Methods in Structures

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There is a slight technical problem in adding methods to dependent structures. The "self" parameter of a method in a binding needs to know the final signature, since the method may want to refer to methods to its right through self. So the usual left-to-right checking of structures and signatures does not quite work. I don't think we want to make the entire binding recursive. So here is a left-to-right solution that accounts for mutual method access through self.

A *structure* {B} contains a dependent *binding* B, and a *signature* {D} contains a dependent *declaration* D. We keep track at all times of the final declaration, via the judgment $\Gamma \vdash B : D \int D'$, meaning that B has declaration D, but must still be "integrated" with D'. Eventually we prove $\Gamma \vdash B : D \int ()$ and we are done.

See [Harper Lillibridge 1993] for the notation and the other necessary rules. Here $\tilde{D}$ is the label-stripping function, and $\overline{\xi}$ is a vector of variables/labels, where each ξ is a type or term variable/label.

Bindings: B
Structures: {B}
Declarations: D
Signatures: {D}

$\Gamma \vdash A :: K$	type A has kind K
$\Gamma \vdash a : A$	term a has type A
$\Gamma \vdash B : D \int D'$	binding B has declaration D, pending D'
$\Gamma \vdash \{B\} : \{D\}$	structure {B} has signature {D}

$x \div A$ method x has result type A (used in contexts and declarations)

(Empty binding)

$$\frac{\Gamma \vdash D}{\Gamma \vdash () : () \int D}$$

(Type binding)

$$\frac{\Gamma \vdash B : D \int b \triangleright X::K, D' \qquad \Gamma, \tilde{D} \vdash A::K \qquad X \notin dom(\Gamma, \tilde{D})}{\Gamma \vdash B, b \triangleright X::K=A \ : \ D, b \triangleright X::K \int D'}$$

(Term binding)

$$\frac{\Gamma \vdash B : D \int b \triangleright x:A, D' \qquad \Gamma, \tilde{D} \vdash a:A \qquad x \notin dom(\Gamma, \tilde{D})}{\Gamma \vdash B, b \triangleright x:A=a \ : \ D, b \triangleright x:A \int D'}$$

(Method binding) where S = {D, b▷x÷A{ $\overline{\xi}$ }, D'}

$$\frac{\Gamma \vdash B : D\int b\triangleright x\div A, D' \qquad \Gamma, \tilde{D}, y:S \vdash a:(A\{\overline{y.\xi}\}) \qquad x,y\notin dom(\Gamma,\tilde{D}) \quad \overline{\xi}\in dom(\tilde{D})}{\Gamma \vdash B, b\triangleright x\div A\{\overline{\xi}\}=\varsigma(y:S)a \ : \ D, b\triangleright x\div A\{\overline{\xi}\}\int D'}$$

(Structure)

$$\frac{\Gamma \vdash B : D\int()}{\Gamma \vdash \{B\}:\{D\}}$$

(Method invocation)

$$\frac{\Gamma \vdash a : \{b\triangleright x\div A\}}{\Gamma \vdash a.b : A}$$

(Method override) where S = {D, b▷x÷A{ $\overline{\xi}$ }, D'}

$$\frac{\Gamma \vdash a : S \qquad \Gamma, \overline{D}, y:S \vdash a':A\{\overline{y.\xi}\} \qquad y\notin dom(\Gamma,\overline{D}) \quad \overline{\xi}\in dom(\tilde{D})}{\Gamma \vdash a.b:=\varsigma(y:S)a' : S}$$

The substitution A{ $\overline{y.\xi}$ } in the (Method binding) rule needs some explanations. At first I wrote:

(Method binding 0) where S = {D, b▷x÷A, D'}

$$\frac{\Gamma \vdash B : D\int b\triangleright x\div A, D' \qquad \Gamma, \overline{D}, y:S \vdash a:A \qquad x,y\notin dom(\Gamma,\overline{D}) \quad y\notin A}{\Gamma \vdash B, b\triangleright x\div A=\varsigma(y:S)a \ : \ D, b\triangleright x\div A\int D'}$$

Where the restriction y∉A is similar to the usual restriction for the dot notation in function result types. But, according to (Method binding 0), the following does not typecheck (I am abbraviating $\xi\triangleright\xi$ as ξ):

{X::Type = Int, z÷X = ς(y:{X::Type, z÷X}) y.z} : {X::Type = Int, z÷X}

However, we know that the current D in (Method binding 0) is a prefix of the final S, which is the type of y. Hence y.X is really the same as X in the current context., for any X declared in D. This is what (Method binding) is saying, and the example above is then typeable.

By the way, for a similar situation [Harper Lillibridge 1993] uses the following rules:

$$\frac{\Gamma,x:A \vdash a:A'}{\Gamma \vdash \lambda(x:A)a : \Pi(x:A)A'}$$

$$\frac{\Gamma \vdash a':\Pi(x:A)A' \qquad \Gamma \vdash a:A \qquad x\notin A'}{\Gamma \vdash a'(a) : A'}$$

I don't quite understand why the side condition is placed on elimination, and not on introduction. Without subsumption, if x occurs in A' then λ(x:A)a is unusable, and we are only delaying the error messages. Subsumption can eliminate occurrences of x in A', but is this really useful?.