Abstract Machines of Systems Biology

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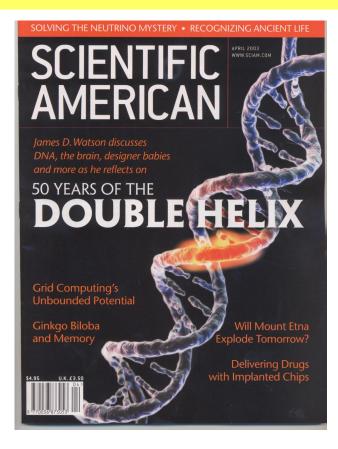
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50 Years of <u>Molecular Cell Biology</u>

- Genes are made of DNA
 - Store digital information as sequences of 4 different nucleotides
 - Direct protein assembly through RNA and the Genetic Code
- Proteins (>10000) are made of amino acids
 - Process signals
 - Activate genes
 - Move materials
 - Catalyze reactions to produce substances
 - Control energy production and consumption
- Bootstrapping still a mystery
 - DNA, RNA, proteins, membranes are today interdependent. Not clear who came first
 - Separation of tasks happened a long time ago
 - Not understood, not essential



Towards <u>Systems Biology</u>

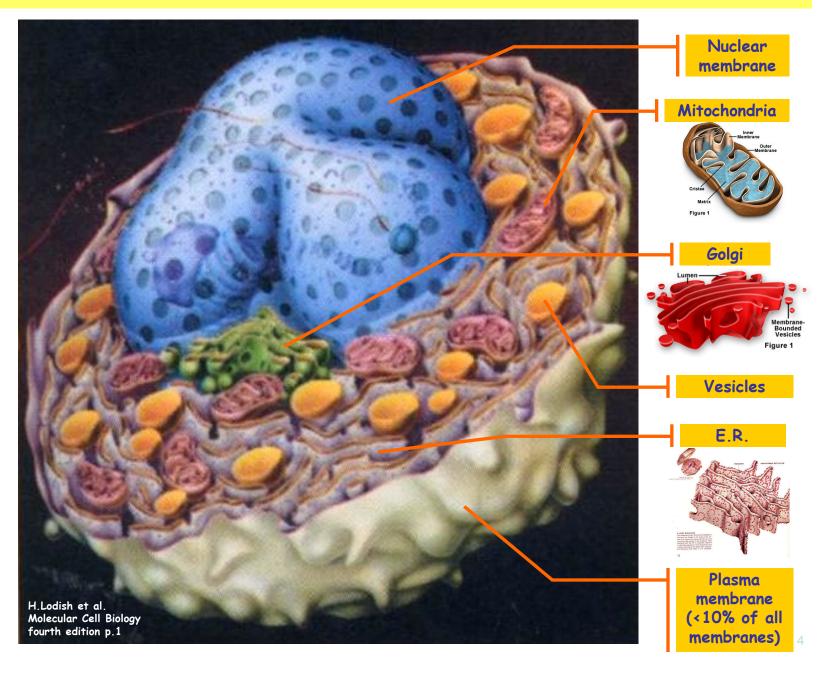
- Biologists now understand many of the cellular components
 - A whole team of biologists will typically study a single protein for years
 - Reductionism: understand the components in order to understand the system
- But this has not led to understand how "the system" works
 - Behavior comes from complex patterns of interactions between components
 - Predictive biology and pharmacology still rare
 - Synthetic biology still unreliable
- New approach: try to understand "the system"
 - Experimentally: massive data gathering and data mining (e.g. Genome projects)
 - Conceptually: modeling and analyzing networks (i.e. interactions) of components
- What kind of a system?
 - Just beyond the basic chemistry of energy and materials processing...
 - Built right out of digital information (DNA)
 - Based on information processing for both survival and evolution
 - Highly concurrent
- Can we fix it when it breaks?
 - Really becomes: How is information structured and processed?

Structural Architecture



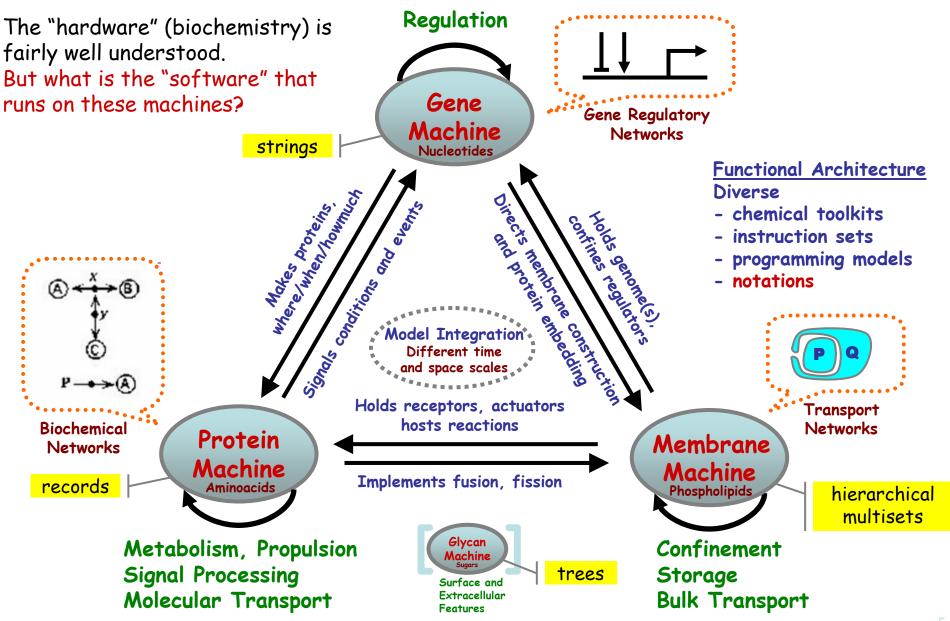
(10~100 trillion in human body)

Membranes everywhere





Abstract Machines of Systems Biology



Storing Processes

- Today we represent, store, search, and analyze:
 - Gene sequence data
 - Protein structure data
 - Metabolic network data
 - Signaling pathway data

Cellular Abstractions: Cells as Computation Regev&Shapiro NATURE vol 419, 2002-09-26, 343

- How can we represent, store, and analyze *biological processes*?
 - Scalable, precise, dynamic, highly structured, maintainable representations for *systems biology*.
 - Not just huge lists of chemical reactions or differential equations.
- In computing...

...

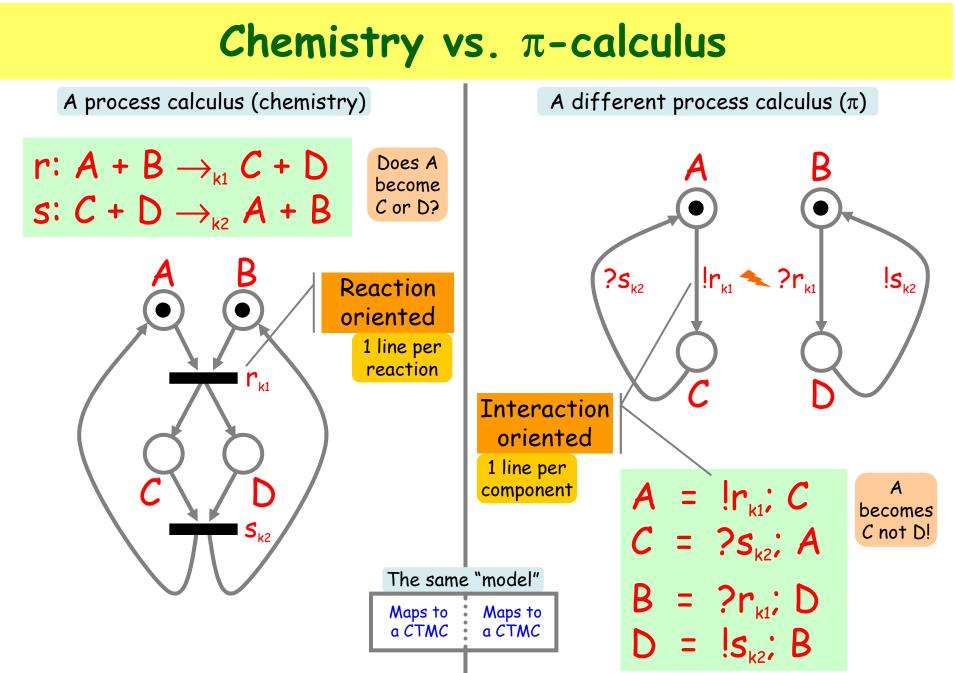
- There are well-established scalable representations of dynamic reactive processes.
- They look more or less like little, mathematically based, programming languages.

Reactive Systems

- Modeling biological systems
 - Not as continuous systems (often highly nonlinear)
 - But as discrete reactive systems; abstract machines where:
 - States represent situations
 - Event-driven transitions between states represent dynamics
 - The adequacy of describing (discrete) complex systems as reactive systems has been argued convincingly [Harel]
- Many biological systems exhibit features of reactive systems:
 - Discrete transitions between states
 - Deep layering of abstractions ("steps" at multiple levels)
 - Complexity from combinatorial interaction of simple components
 - High degree of concurrency and nondeterminism
 - "Emergent behavior" not obvious from part list
- Still, needs quantitative semantics
 - Stochastic, hybrid, etc. to talk about *rates* (and geometry).

Methods

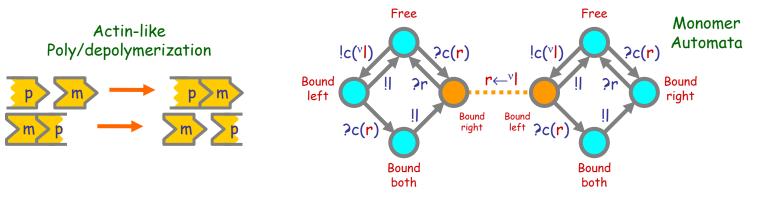
- Model Construction (writing things down precisely)
 - Formalizing the notations used in systems biology.
 - Formulating description languages.
 - Studying their kinetics (semantics).
- Model Validation (using models for postdiction and prediction)
 - Simulation from compositional descriptions
 - Stochastic: quantitative concurrent semantics.
 - Hybrid: discrete transitions between continuously evolving states.
 - "Program" Analysis
 - Control flow analysis
 - Causality analysis
 - Modelchecking
 - Standard, Quantitative, Probabilistic



A Petri-Net-like representation. Precise and dynamic A compositional graphical representation (precise, but not modular, scalable, or maintainable. A compositional graphical representation (precise, dynamic *and* modular) and the corresponding calculus.

Why π -calculus, in particular

- Well studied, compact, precise, and general
 - A "programming language" first, a mathematical model second
 - Syntax (configurations): P ::= 0 | P+P | P|P | ?n(n).P | !n(n).P | (vn)P | *P
 - Semantics (reactions): $(P' + !n(m).P) | (Q' + ?n(m').Q) \rightarrow P | Q\{m' \leftarrow m\}$
- Binary interactions
 - I.e., "collisions"
- Reactive and compositional
 - Each subsystem is a separate (composition of) nondeterministic automata interacting with the environment (more automata)
- Dynamic network evolution and species evolution
 - Each subsystem can create fresh connections or spawn new subsystems
- Compact description of combinatorics (like any programming language)
 - $(Bit_1 | Bit_2 | ... | Bit_n)$ size-n description, where Bit is a 2-state subsystem
 - E.g. a protein with with 2ⁿ phosphorilation configurations (i.e. "different chemical species")
- Complexation/polymerization
 - The most characteristic feature of π -calculus (fresh names) models "sticking"



Stochastic π -calculus

- A quantitative variant of π -calculus:
 - Channels have stochastic "firing" rates with exponential distribution.
 - Nondeterministic choice becomes *stochastic race*.
 - Cuts down to CTMCs (Continuous Time Markov Chains) in the finite case (not always). Then, standard analytical tools are applicable.
 - Can be given friendly automata-like scalable graphical syntax (Andrew Phillips et al.).
 - Is directly executable (e.g. via the Gillespie algorithm from physical chemistry).
 - Is analyzable (large body of literature, at least in the nonstochastic case).

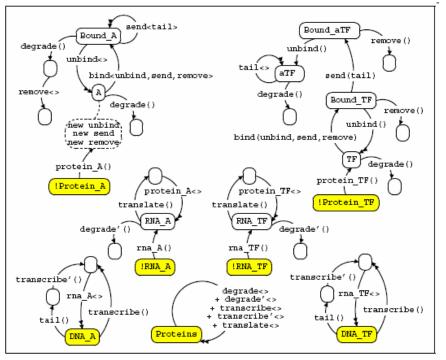


Figure 2. Regulating Gene Expression by Positive Feedback [9]

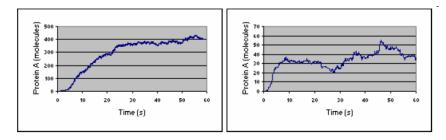
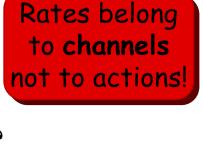
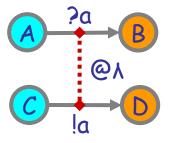


Figure 3. Protein A molecules v.s. time in presence (left) and absence (right) of TF A.Phillips, L.Cardelli. BioConcur'04.

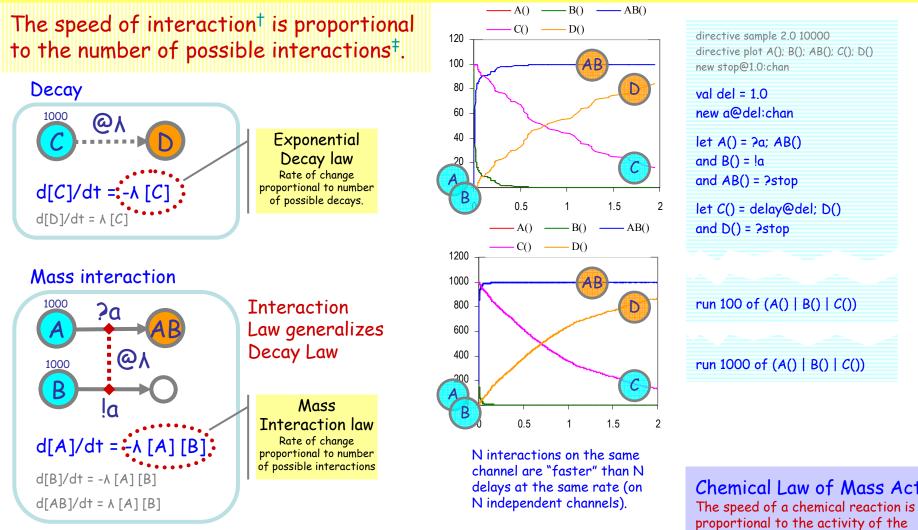
The Rate of What?

- In chemistry:
 - Each reaction involves 2 molecules, and each reaction has a **rate**. Rates belongs to **reactions**. Molecules do *not* have rates.
- In process algebras:
 - Should rates belong to:
 - each individual action? only outputs? delays only?
 - The rate of a synchronization of two actions should be the:
 - max? product? undefined if different? infinite (except for delays)?
 - All that has been tried.
- We go back to chemistry
 - Rates belong to **channels**. (This is called the "biochemical" stochastic π -calculus by Priami-Regev-Shapiro-Silverman)
- Issues:
 - Multiple activities on the same channel (concentrations of molecules involved in a reaction: mass action law of chemistry).
 - Choices between different channels (molecules involved in multiple reactions: still standard chemistry).
 - In biochemistry, rates of homodimerization (a molecule can interact with a copy of itself, but not with itself).





Law of Mass Interaction



[†] speed of interaction = apparent rate (formally definable) = number of interactions over time

[†] not proportional to the number of interacting processes! [P] is the number of processes P (this is informal; it is only meaningful for a set of processes offering a given action, but a set of such processes can be counted and plotted)

Each C has A chances to decay per second (no matter how many other Cs there are), but each A has [B]*A chances to interact per second: it depends on how many Bs there are.

Chemical Law of Mass Action reacting substances.

(Activity = concentration, for wellstirred aqueous medium)

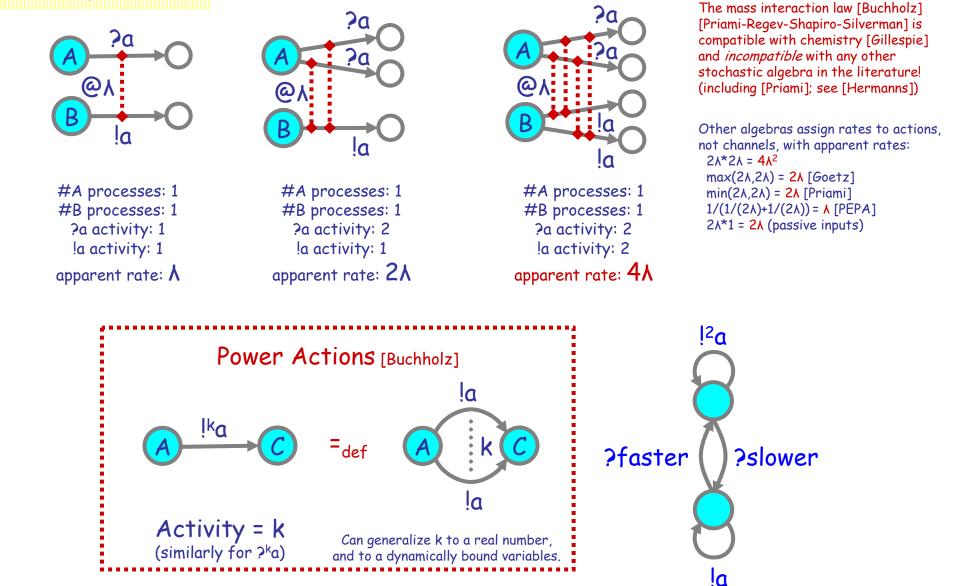
(Concentration = number of moles per liter of solution)

(Mole = 6.022141×10²³ particles)

Activity and Apparent Rate

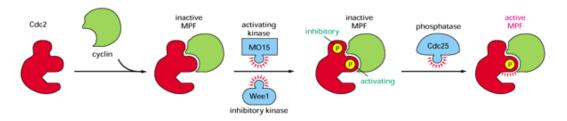
stochastic algebras disagree!

The speed of interaction is proportional to the number of possible interactions.

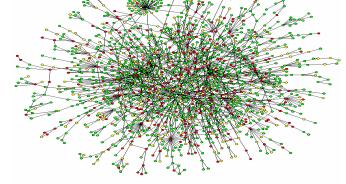


1. The Protein Machine

- Complex folded-up shapes that:
 - Fit together, dock, undock.
 - Excite/unexcite, warp each other.
 - Bring together, catalyze, transform materials.
 - Form complex aggregates and networks.



- Mapping out such networks:
 - In principle, it's "just" a very large set of chemical equations.
 - Notations have been developed to summarize and abstract.



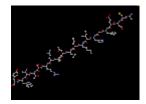
An actual molecular interaction network. (Nodes are distinct protein kinds, arcs mean that two kinds of proteins interact.)

Very close to

the atoms.

Protein Structure

Primary

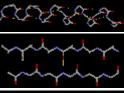


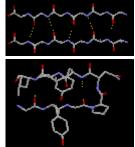
The 20 Aminoacids



Tryptophan

Secondary





Alpha Helix, Beta Sheet

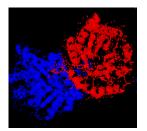
Tertiary



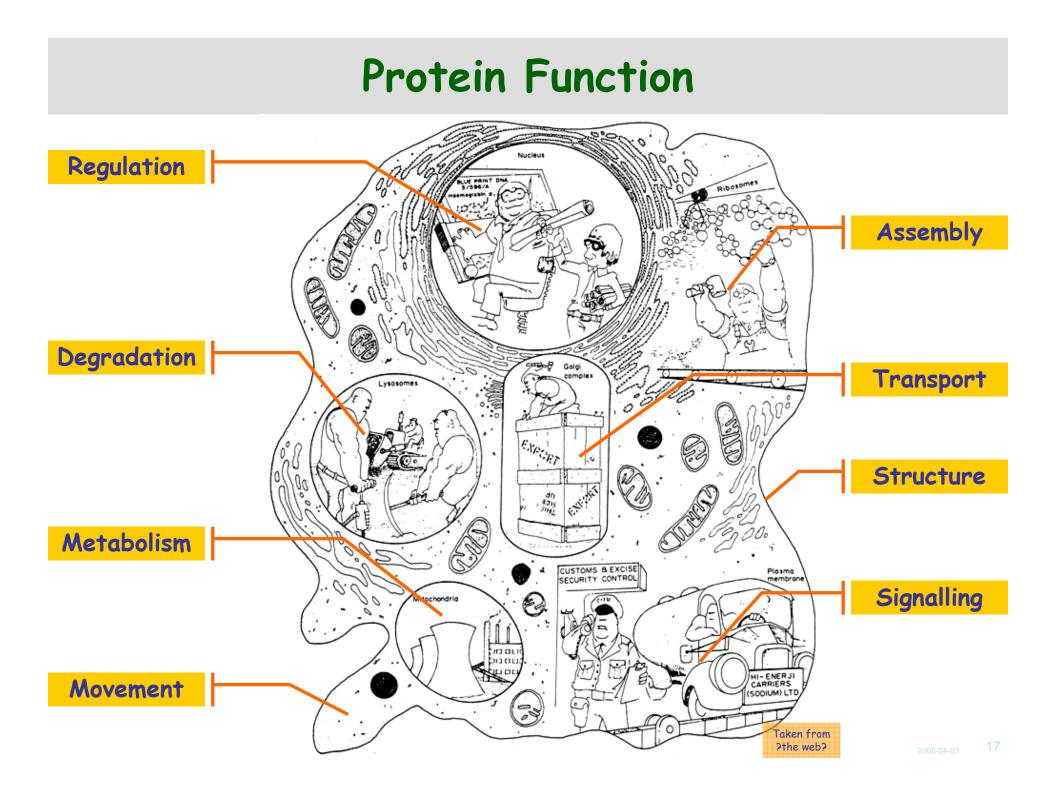


Green Fluorescent Protein

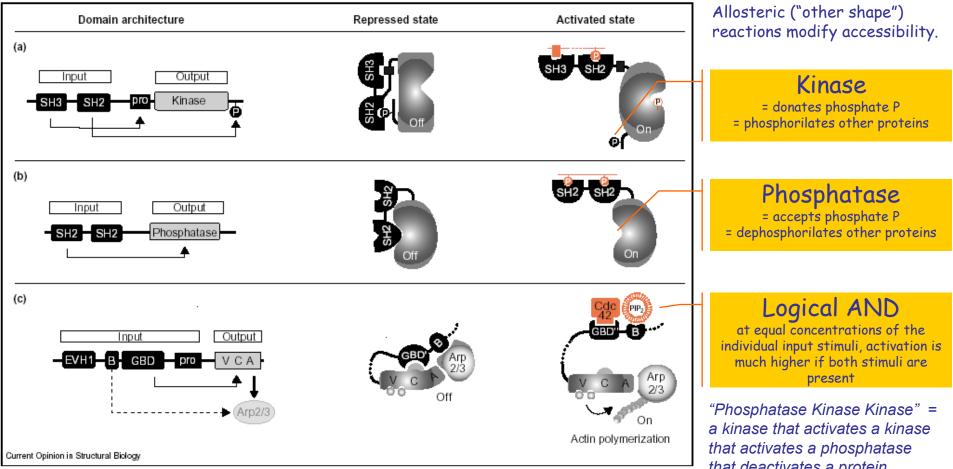
Quaternary



Triose Phosphate Isomerase



Some Allosteric Switches



Domain architecture and autoinhibitory interactions in modular switch proteins. (a) Src family kinases contain N-terminal SH3 and SH2 domains, and a kinase domain flanked by intramolecular SH3-binding and SH2-binding sites (when the C-terminal motif tyrosine is phosphorylated by Csk). The crystal structures of several family members show that both intramolecular domain interactions function in concert to lock the kinase in an inactive conformation. Activating stimuli (red) include external SH2 or SH3 ligands. After initial activation, the kinase is maintained in an active state by autophosphorylation of its activation loop. (b) SHP-2 phosphatase contains two SH2 domains and a phosphatase domain. The crystal structure of the phosphatase

shows that the N-terminal SH2 domain participates in an autoinhibitory interaction that directly blocks the phosphatase active site. Binding of external SH2 ligands activates by disrupting the autoinhibitory interaction. (c) N-WASP contains an Enabled VASP homology 1 (EVH1) domain, a B motif, a GBD, a proline-rich segment (pro) and an output region (VCA) that alone binds the Arp2/3 complex and stimulates its actin nucleation activity. The B and GBD motifs are required to repress activity and, by current models, are thought to participate in intracomplex interactions (only the structure of the GBD intramolecular complex for WASP is known). GTP-bound Cdc42 and PIP₂ synergistically activate N-WASP.

that deactivates a protein.

Humans have the same number of modular protein domains (building blocks) as worms, but twice the number of multi-domain proteins.



MIM: Molecular Interaction Maps (Kohn)

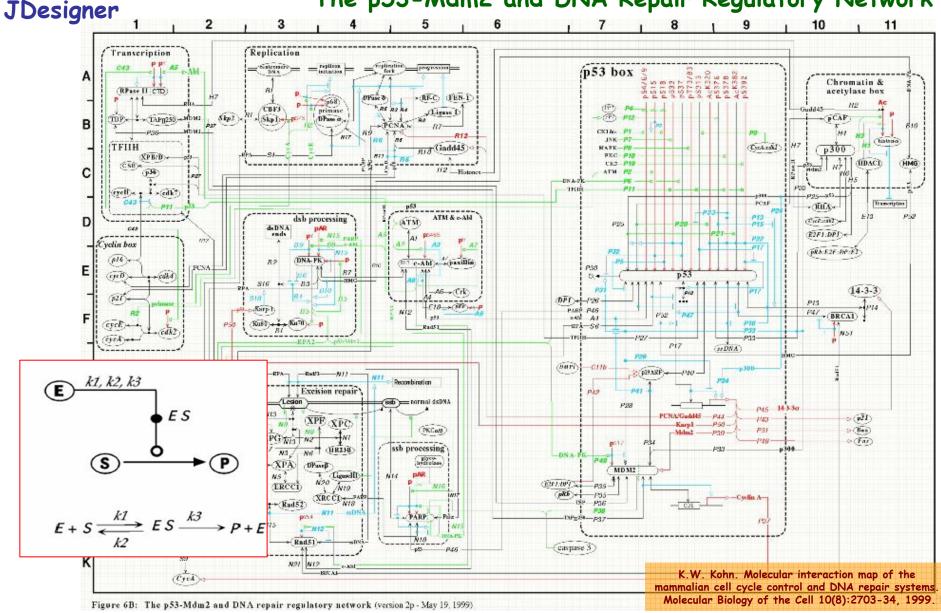
()	$\rightarrow \odot$ The double-arrowed line indicates that proteins A and B can bind to each other. The "node" placed on the line	(A) → (B)	Stoichiometric conversion of A into B.
	 represents the A:B complex. Asymmetric binding where protein A donates a peptide that binds to a receptor site or pucket on protein B. 	Cytosol nucleus	Transport of A from cytosol to nucleus. The node represents A after it has been transported into the nucleus.
⊗ < x	$ \begin{array}{l} \Rightarrow \textcircled{B} \\ y \end{array} \begin{array}{l} \text{Representation of multimolecular complexes: } x \text{ is } \textbf{A:B;} \\ y \text{ is } (\textbf{A:B}): \textbf{C}. \\ \end{array} \begin{array}{l} \text{This notation is extensible to any number} \\ \text{of components in a complex.} \end{array} $		Formation of a homodimer. Filled circle on the righ represents another copy of A . The node on the line represents the homodimer A : A .
С Р—•	 Covalent modification of protein A. The single-arrowed Ine indicates that A can exist in a phosphorylated state. The node represents the phosphorylated species. 	x z y	z is the combination of states defined by x and y . Enzymatic stimulation of a reaction.
Phitase	$ \begin{array}{c} \mathbf{P} \\ \hline \\ \mathbf{P} \\ \mathbf{P}$		General symbol for stimulation. A bar behind the arrowhead signifies necessity. General symbol for inhibition.
	 Proteolytic cleavage at a specific site within a protein. 	حمال المحال ا	Shorthand symbol for transcriptional activation. Shorthand symbol for transcriptional inhibition.
1		Ø	Degradation products

- ion of states defined by x and y.
- ation of a reaction.
- or stimulation. arrowhead signifies necessity.
- or inhibition.
- for transcriptional activation.
 - for transcriptional inhibition.
 - ucts

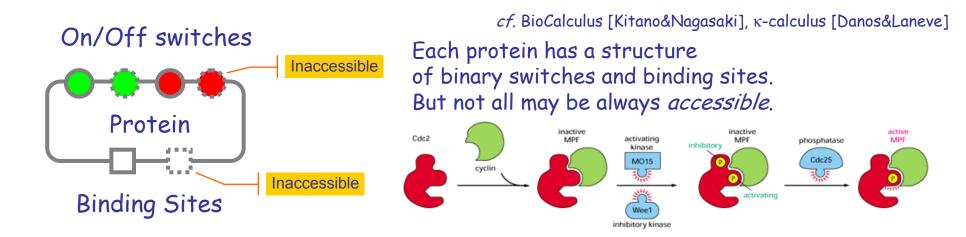
Molecular Interaction Maps

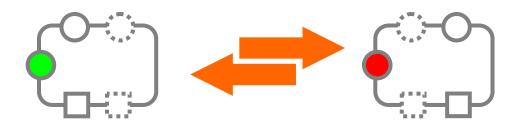
http://www.cds.caltech.edu/~hsauro/index.htm

The p53-Mdm2 and DNA Repair Regulatory Network



The Protein Machine "Instruction Set"



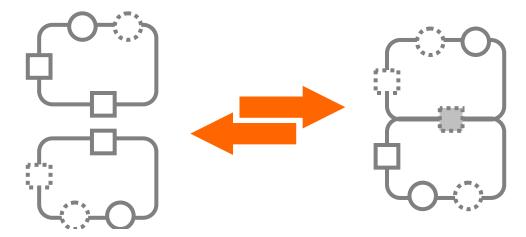


Switching of accessible switches.

- May cause other switches and

binding sites to become (in)accessible.

- May be triggered or inhibited by nearby specific proteins in specific states.



Binding on accessible sites.

- May cause other switches and

binding sites to become (in)accessible.

- May be triggered or inhibited by nearby specific proteins in specific states.

Notations for the Protein Machine

- Stochastic π -Calculus
 - Priami (following Hillston's PEPA) formalizes a stochastic version of p-calculus where channels have communication *rates*.
- BioSPi
 - Regev-Shapiro-Silverman propose modeling chemical interactions (exchange of electrons and small molecules) as "communication".
 - Standard stochastic simulation algorithms (Gillespie) can be used to run in-silico experiments.
 - Complex formation is encoded via p-restriction.
- PEPA
 - Calder Gilmore and Hillston model the ERK pathway.
- k-calculus
 - Danos and Laneve (following Kitano's BioCalculus) define a calculus where complex formation is primitive.
- (Stochastic) Petri Nets
 - S.Reddy'94 modeling pathways.
 - Srivastava Perterson and Bentley analyze and simulate E.coli stress response circuit.

- Bio State Charts
 - Harel uses State Charts to model biological interactions via a semi-graphical FSM notation.
- Pathway Logic
 - Talcott-Eker-Knapp-Lincoln use term-rewriting.
- BioCham
 - ChabrierRivier-Fages-Soliman use term-rewriting and CLT modelchecking.
- Kohn Diagrams, Kitano Diagrams
- SBML (Systems Biology Markup Language)
 - XML dialect for MIM's:
 - Compartments (statically nested)
 - Reagents with concentrations
 - Reactions with various rate laws
 - Read and written by many tools via the Systems Biology Workbench protocol

MAPK Cascade

<u>Ultrasensitivity in the mitogen-activated protein cascade</u>, Chi-Ying F. Huang and James E. Ferrell, Jr., 1996, <u>Proc. Natl. Acad. Sci. USA</u>, 93, 10078-10083.

Biochemistry: Huang and Ferrel	I	Proc. Natl. Acad. Sci. USA 93 (1996)						
Table 2. Predicted Hill coefficients for MAP kinase cascade components: Varying the assumed K _m values								
		Range of ef	Range of effective Hill coefficients (nH)					
	Range of assumed K_m		predicted for					
Reaction	values	MAPKKK	MAPKK	MAPK				
1. MAPKKK \rightarrow MAPKKK*	60-1500 nM	1.0	1.7	4.9				
2. MAPKKK $* \rightarrow$ MAPKKK	60 1500 nM	1.0	1.7	4.9				
3. MAPKK \rightarrow MAPKK-P	60-1500 nM	1.0	1.3-2.3	4.0 - 5.1				
4. MAPKK-P → MAPKK	60–1500 nM	1.0	1.5 - 1.9	3.6-6.7				
5. MAPKK-P \rightarrow MAPKK-PP	60–1500 nM	1.0	1.3-2.4	3.8-5.2				
6. MAPKK-PP \rightarrow MAPKK-P	60–1500 nM	1.0	1.7-1.8	4.1-6.4				
7. MAPK \rightarrow MAPK-P	60–1500 nM (300 nM [†])	1.0	1.7	3.7-6.2				
8. MAPK-P \rightarrow MAPK	60-1500 nM	1.0	1.7	4.3-5.2				
9. MAPK-P \rightarrow MAPK-PP	60–1500 nM	1.0	1.7	3.4 - 6.1				
10. MAPK-PP \rightarrow MAPK-P	60–1500 nM	1.0	1.7	4.7-5.1				

The assumed K_m values for each reaction were individually varied over the ranges shown, with the assumed K_m values for the other nine reactions held constant. The effective Hill coefficients were calculated from the steepness of the predicted stimulus/response curves, as described in the text.

[†]The K_m value for reaction 7 has been measured to be 300 nM for the phosphorylation of a mammalian MAPK by a MAPKK (N. Ahn, personal communication). All of the other K_m values were initially assumed to be 300 nM as well.

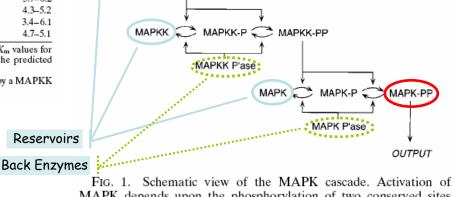
de

KK-P + KK P'ase

[6]

Calculations. Eqs. 1-10 represent the reactions of the MAPK cascade, which are shown schematically in Fig. 1. We have used Goldbeter and Koshland's nomenclature for the rate constants the letter a denotes association, d denotes dissociation without catalysis, and k denotes product formation (11). KKK denotes MAPKKK: KK denotes MAPKK; and K denotes MAPK.

KKK + EI
$$\stackrel{a_1}{\longrightarrow}$$
 KKK* E1 $\stackrel{k_1}{\longrightarrow}$ KKK* + E1[1]KK-PP + K $\stackrel{a_7}{\longrightarrow}$ KK-PP + K-P[7]KKK* + E2 $\stackrel{a_2}{\longrightarrow}$ KKK+E2 $\stackrel{a_2}{\longrightarrow}$ KKK+E2[2]K-P + K P'ase $\stackrel{a_3}{\longrightarrow}$ K-P + K P'ase[8]KK + KKK* $\stackrel{a_3}{\longrightarrow}$ KK-P + KKK*[3]K-P + K P'ase $\stackrel{a_3}{\longrightarrow}$ K-P + K P'ase[8]KK-P + KK P'ase $\stackrel{a_4}{\longrightarrow}$ KK-P + KK P'ase[4]K-P + K P'ase $\stackrel{a_9}{\longrightarrow}$ K-PP + KK-PP[9]KK-P + KKK* $\stackrel{a_5}{\longrightarrow}$ KK-P + KK P'ase[4]K-PP + K P'ase $\stackrel{a_{10}}{\longrightarrow}$ KK-PP + KP'ase[10]KK-P + KKK* $\stackrel{a_5}{\longrightarrow}$ KK-P + KKK*[5] $\stackrel{k_{10}}{\longrightarrow}$ K-P + K P'ase[10]



10 chemical

reactions

MAPKKK*

INPUT (E1)

E2

МАРККК 🧲

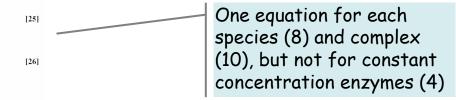
FIG. 1. Schematic view of the MAPK cascade. Activation of MAPK depends upon the phosphorylation of two conserved sites [Thr-183 and Tyr-185 in rat p42 MAPK/Erk2 (4, 5)]. Full activation of MAPKK also requires phosphorylation of two sites [Ser-218 and Ser-222 in mouse Mek-1/MKK1 (6–10)]. Detailed mechanisms for the activation of various MAPKKKs (e.g., Raf-1, B-Raf, Mos) are not yet established; here we assume that MAPKKKs are activated and inactivated by enzymes we denote E1 and E2. MAPKKK* denotes activated MAPKKK. MAPKK-P and MAPKK-PP denote singly and doubly phosphorylated MAPKK, respectively. MAPK-P and MAPK-PP denote singly and doubly phosphorylated MAPK. P'ase denotes phosphatase.

As 18 Ordinary Differential Equations Plus 7 conservation equations

$\frac{d}{dt}[KKK] = -a_1[KKK][E1] + d_1[KKK \cdot E1]$	-
+ $k_2[KKK^* \cdot E2]$	[11]
$\frac{d}{dt}[KKK\cdotE1] = a_1[KKK][E1] - (d_1 + k_1)[KKK\cdotE1]$	[12]
$\begin{split} &\frac{d}{dt} [KKK^*] = -a_2 [KKK^*] [E2] + d_2 [KKK^* \cdot E2] \\ &+ k_1 [KKK \cdot E1] + (k_3 + d_3) [KK \cdot KKK^*] - a_3 [KKK^*] \\ &+ (k_5 + d_5) [KK \cdot P \cdot KKK^*] - a_3 [KK \cdot P] [KKK^*] \end{split}$	[13]
$\frac{d}{dt}[KKK^{*}E2] = a_2[KKK^{*}][E2] - (d_2 + k_2)[KKK^{*}E2]$] [14]
$\frac{d}{dt}[KK] = -a_3[KK][KKK^*] + d_3[KK \cdot KKK^*]$	
+ $k_4[KK-P \cdot KK P'ase]$	[15]
$\frac{d}{dt}[KK KKK^*] = a_3[KK][KKK^*]$	
$- (d_3 + k_3)[KK \cdot KKK^*]$	[16]
$\frac{d}{dt}[KK-P] = -a_4[KK-P][KK P'ase] + d_4[KK-P\cdotKK P'ase] + k_3[KK \cdot KKK^*] + k_6[KK-PP \cdot KK P'ase]$	ise]
+ $d_{s}[KK-P \cdot KKK^*] - a_{s}[KK-P][KKK^*]$	[17]
+ $d_{5}[KK-P \cdot KKK^{*}] - a_{5}[KK-P][KKK^{*}]$	[17]
$\frac{d}{dt}[KK-PKK P'ase] = a_4[KK-P][KK P'ase]$	
$- (d_4 + k_4)[KK - P \cdot KK P' ase]$	[18]
$\frac{d}{dt}[KK-P\cdot KKK^*] = a_5[KK-P][KKK^*]$	
$- (d_5 + k_5)[KK - P \cdot KKK^*]$	[19]
$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} \left[\mathrm{KK}\text{-}\mathrm{PP} \right] &= \mathrm{k}_{3} \left[\mathrm{KK}\text{-}\mathrm{P}\text{K}\mathrm{KK}^{*} \right] - \mathrm{a}_{6} \left[\mathrm{KK}\text{-}\mathrm{PP} \right] \left[\mathrm{KK} \ \mathrm{P}' \mathrm{ase} \right. \\ &+ \mathrm{d}_{6} \left[\mathrm{KK}\text{-}\mathrm{PP} \cdot \mathrm{KK} \ \mathrm{P}' \mathrm{ase} \right] - \mathrm{a}_{7} \left[\mathrm{KK}\text{-}\mathrm{PP} \right] \right] \\ &+ \mathrm{d}_{7} + \mathrm{k}_{7} \left[\mathrm{K} \cdot \mathrm{KK}\text{-}\mathrm{PP} \right] \\ &+ \mathrm{d}_{7} + \mathrm{k}_{7} \left[\mathrm{K} \cdot \mathrm{KK}\text{-}\mathrm{PP} \right] \\ &+ \mathrm{d}_{9} + \mathrm{k}_{9} \left[\mathrm{K} - \mathrm{P} \cdot \mathrm{KK}\text{-}\mathrm{PP} \right] \end{aligned}$	
$-a_{9}[K-P][KK-PP]$	[20]
$\label{eq:kk-PP} \begin{split} \frac{d}{dt} \left[KK\text{-}PP\text{-}KK \ P'ase \right] &= a_6 [KK\text{-}PP] [KK \ P'ase] \\ &- (d_6 + K_6) [KK\text{-}PP\text{-}KK \ P'ase] \end{split}$	[21]
$\frac{d}{dt}[K] = -a_7[K][KK-PP] + d_7[K\cdot KK-PP]$	
+ $k_{8}[K-P \cdot K P'ase]$	[22]
$\frac{d}{dt} [\mathbf{K} \cdot \mathbf{K} \mathbf{K} - \mathbf{PP}] = \mathbf{a}_7 [\mathbf{K}] [\mathbf{K} \mathbf{K} - \mathbf{PP}] - (\mathbf{d}_7 + \mathbf{k}_7) [\mathbf{K} \cdot \mathbf{K} \mathbf{K} - \mathbf{PP}]$?] [23]

$\frac{d}{dt}[K-P] = k_7[K\cdot KK-PP] - a_8[K-P][K P'ase] + d_8[K-P \cdot KP'ase] - a_8[K-P][KK-PP]$	
$+ d_{9}[K \cdot P \cdot KK \cdot PP] + k_{10}[K \cdot PP \cdot KP' ase]$	[24]
$\frac{d}{dt} [K-P \cdot K P' ase] = a_8 [K-P] [K P' ase]$	
$- (d_8 + k_8)[K - P \cdot K P' ase]$	[25]
$\frac{d}{dt}[K-P\cdot KK-PP] = a_9[K-P][KK-PP]$	
$- (d_9 + k_9)[K - P \cdot KK - PP]$	[26]
$\frac{d}{dt}[K-PP] = -a_{10}[K-PP][K P'ase]$	
+ $d_{10}[K-PP \cdot KP'ase]$ + $k_9[K-P \cdot KK-PP]$	[27]
$\frac{d}{dt}[K-PP\cdot K P'ase] = a_{10}[K-PP][K P'ase]$	
$- (d_{10} + k_{10})[K-PP \cdot K P'ase]$	[28]
$[E1_{tot}] = [E1] + [KKK \cdot E1]$	[30]
$[E2_{tot}] = [E2] + [KKK^* \cdot E2]$	[31]
$[KK_{tot}] = [KK] + [KK-P] + [KK-PP] + [KK\cdotKKK]$	[*]
+ $[KK - P \cdot KKK^*]$ + $[KK - P \cdot KK P'ase$?]
+ $[KK-PP \cdot KK P'ase]$	
+ $[KK-PP \cdot K]$ + $[KK-PP \cdot K-P]$	[32]
$[KK P'ase_{tot}] = [KK P'ase] + [KK P'ase KK-P]$	
+ $[KK P'ase \cdot KK-PP]$	[33]
$[K_{tot}] = [K] + [K-P] + [K-PP] + [KK-PP-K]$	
$+ KK-PP \cdot K-P] + [K-P \cdot K P'ase] + [K-PP \cdot K P'ase]$	[34]
$[K P'ase_{tot}] = [K P'ase] + [K P K P'ase]$	1
$+ [K - PP \cdot K P' ase]$	[35]
These equations were solved numerically using the R Kutta-based NDSolve algorithm in Mathematica (W Research, Champaign, IL). An annotated copy of the ematica code for the MAPK cascade rate equations of obtained from J.E.F.	olfram Math-

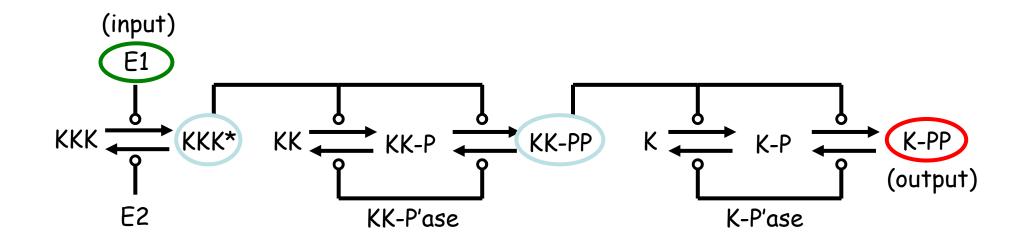
The 10 reactions described above give rise to 18 rate equations.



In addition, there are seven conservation equations (Eqs. 29-35).

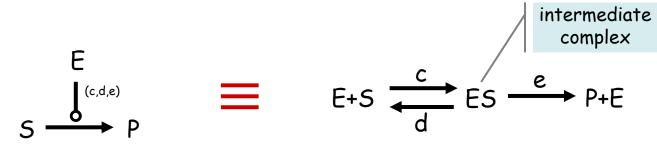
$$[KKK_{tot}] = [KKK] + [KKK^*] + [KKK*E1] + [KKK* \cdot E2] + [KKK* \cdot K] + [KKK* \cdot K-P]$$
in exactly one state
Each molecule
$$Each molecule$$

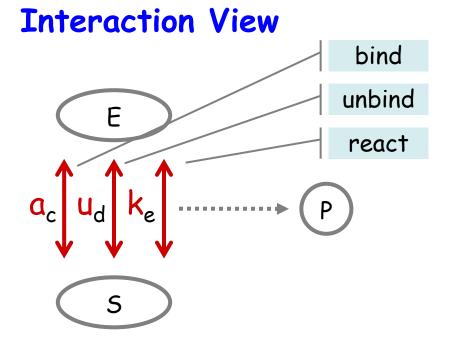
The Circuit



Enzymatic Reactions

Reaction View





private bindings between one S and one E molecule $S() \triangleq new u@d new k@e$ $|a_c(u,k); (!u_d; S() + !k_e; P())$ bind unbind react $E() \triangleq ?a_c(u,k); (?u_d; E() + ?k_e; E())$ $P() \triangleq ...$

As 12 processes (in SPiM)

let KKK() =		and KK_PP() =	
(new u1@d1:Release new k1@r1:React		(new n6@d6:Release new k6@r6:React	
!a1(u1,k1); (do !u1;KKK() or !k1;KKKst()))	[1]substrate	do !a6(u6,k6);	[6]substrate
and KKKst() = (new u2@d2:Release new k2@r2:React do !a2(u2,k2); (do !u2;KKKst() or !k2;KKK()) or ?a3(u3,k3); (do ?u3;KKKst() or ?k3;KKKst())	[2]substrate	() () ne process for each	[7]kinase [9]kinase [4]phtase
or ?a5(u5,k5); (do ?u5;KKKst() or ?k5;KKKst())		or ?a6(u6,k6); (do ?u6;KKPse() or ?k6;KKPse())	[6]phtase
<pre>let E1() =</pre>	[1]enzyme	<pre>let K() = No need for conservation (new u7@ equations: implicit in "choice" !a7(u7,k7)operator in the calculus.</pre>	[7]substrate
?a2(u2,k2); (do ?u2;E2() or ?k2;E2())	[2]enzyme	and K_P() = (new u8@d8:Release new k8@r8:React	
<pre>let KK() = (new u3@d3:Release new k3@r3:React</pre>		new u9@d9:Release new k9@r9:React do !a8(u8,k8); (do !u8;K P() or !k8;K())	[8]substrate
!a3(u3,k3); (do !u3;KK() or !k3;KK_P()))	[3]substrate	or !a9(u9,k9); (do !u9;K_P() or !k9;K_PP()))	[9]substrate
and KK_P() =		and K_PP() =	
(new u4@d4:Release new k4@r4:React new u5@d5:Release new k5@r5:React	[4] and a funct	(new u10@d10:Release new k10@r10:React !a10(u10,k10); (do !u10;K_PP() or !k10;K_P()))	[10]substrate
do !a4(u4,k4); (do !u4;KK_P() or !k4;KK()) or !a5(u5,k5); (do !u5;KK_P() or !k5;KK_PP()))	[4]substrate [5]substrate	and KPse() = do ?a8(u8,k8); (do ?u8;KPse() or ?k8;KPse())	[8]phtase
		or ?a10(u10,k10); (do ?u10;KPse() or ?k10;KPse())	

... and 30 Interaction Channels

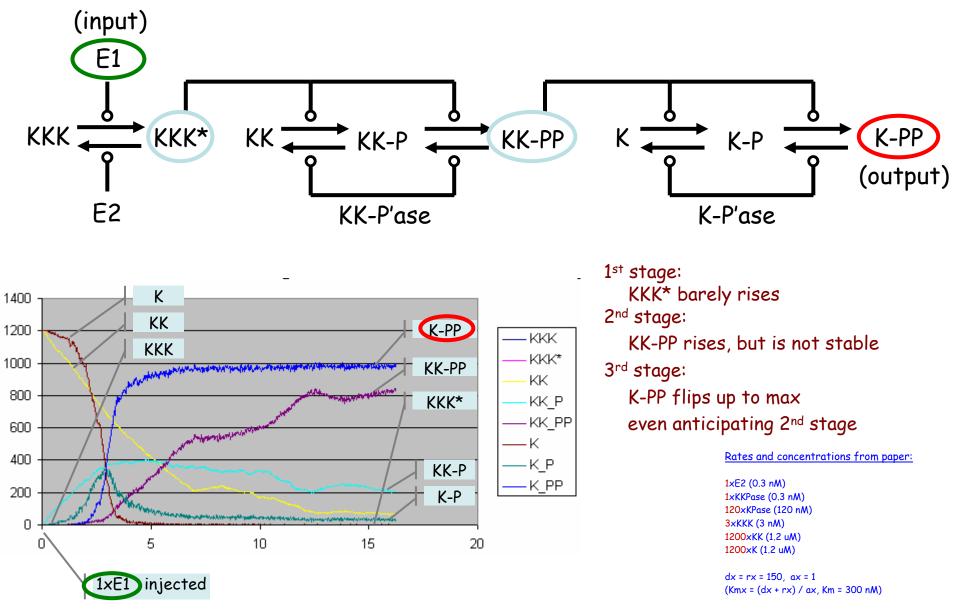
type Release = chan()
type React = chan()
type Bond = chan(Release,React)

```
new a1@1.0:Bond val d1=1.0 val r1=1.0
new a2@1.0:Bond val d2=1.0 val r2=1.0
new a3@1.0:Bond val d3=1.0 val r3=1.0
new a4@1.0:Bond val d4=1.0 val r4=1.0
new a5@1.0:Bond val d5=1.0 val r5=1.0
new a6@1.0:Bond val d6=1.0 val r6=1.0
new a7@1.0:Bond val d7=1.0 val r7=1.0
new a8@1.0:Bond val d8=1.0 val r8=1.0
new a9@1.0:Bond val d9=1.0 val r9=1.0
new a10@1.0:Bond val d10=1.0 val r10=1.0
```

•••

run 100 of KKK() run 100 of KK() run 100 of K() run 1 of E2() run 1 of KKPse() run 1 of KPse() run 1 of E1() a_i(u_i,k_i): release (u_i@d_i) and react (k_i@r_i) channels passed over bond (a_i) channel. (No behavior attached to channels except interaction rate.)

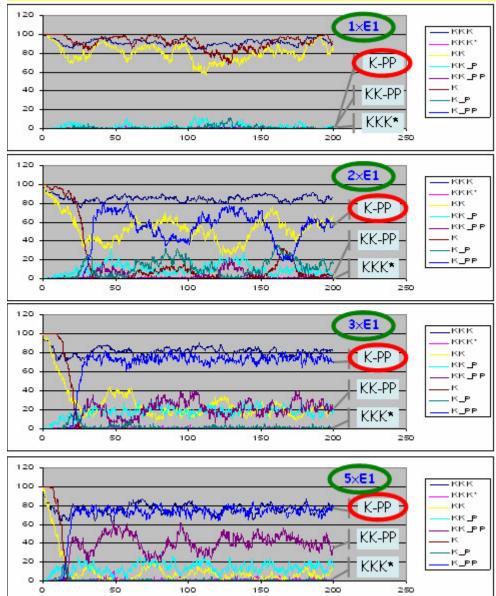
MAPK Cascade Simulation in SPiM

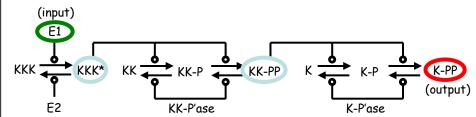


1×E1

2006-04-03

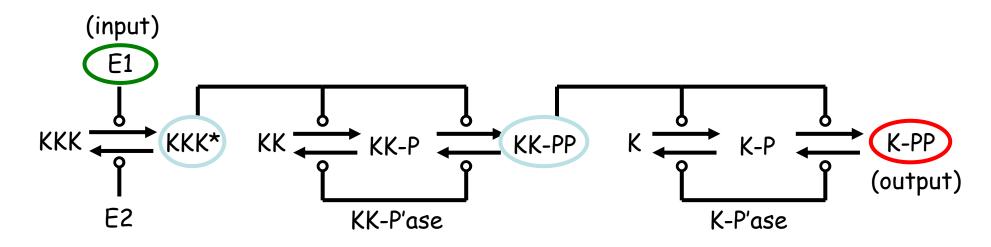
MAPK Cascade Simulation in SPiM

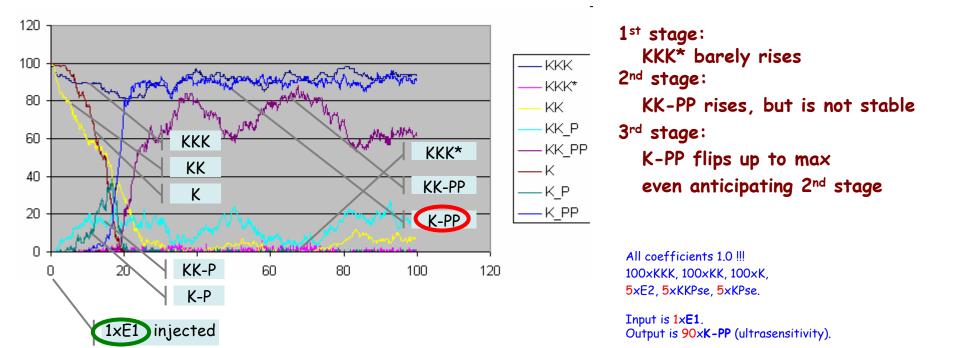




All coefficients 1.0 !!! 100×KKK, 100×KK, 100×K, 13×E2, 13×KKPse, 13×KPse. nxE1 as indicated (1×E1 is not sufficient to produce an output)

MAPK Cascade Simulation in SPiM

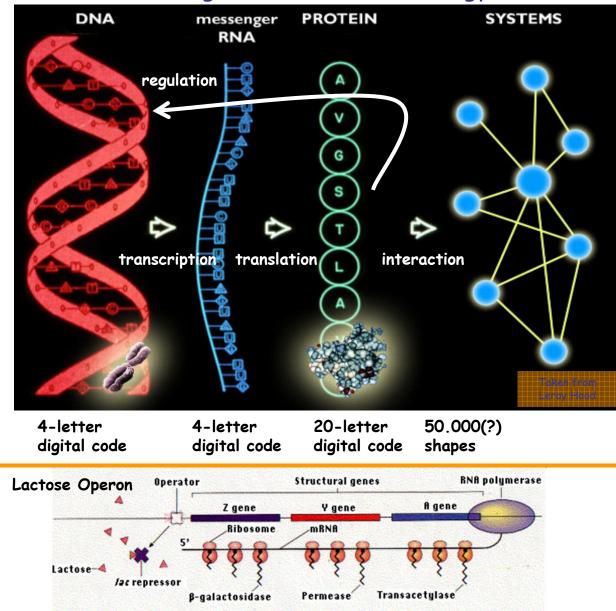


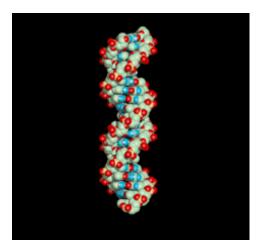


2. The Gene Machine

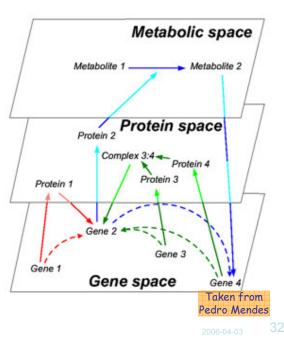
Pretty far from the atoms.

The "Central Dogma" of Molecular Biology



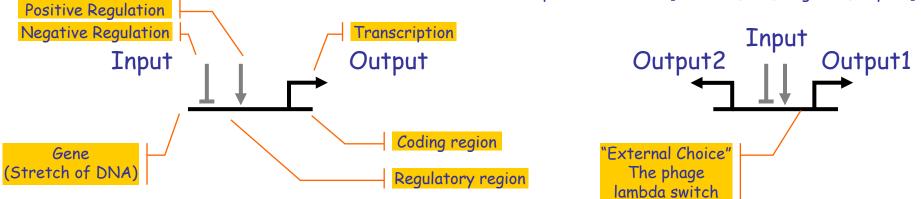


DNA Tutorial



The Gene Machine "Instruction Set"

cf. Hybrid Petri Nets [Matsuno, Doi, Nagasaki, Miyano]

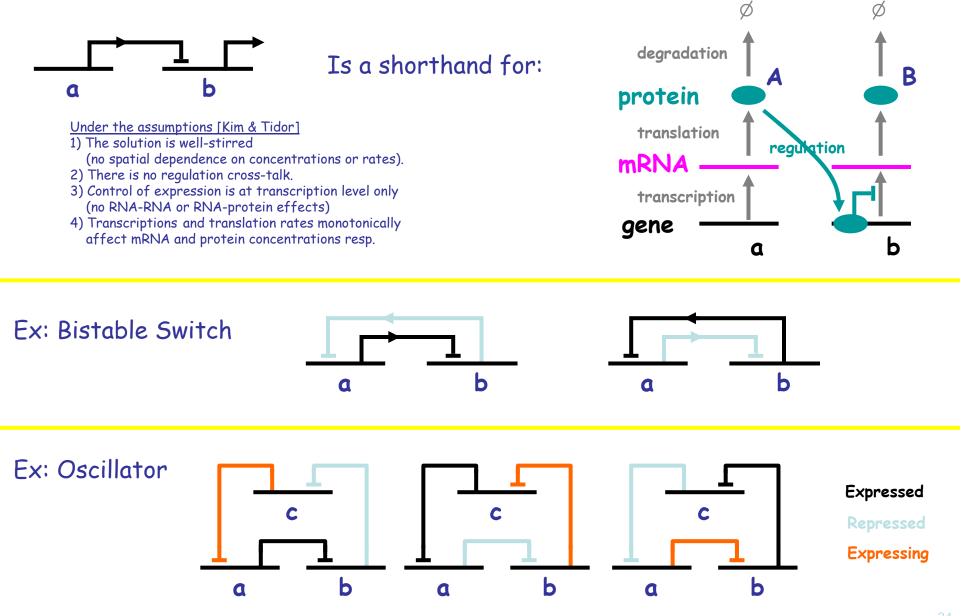


<u>Regulation</u> of a gene (positive and negative) influences transcription. The regulatory region has precise DNA sequences, but not meant for coding proteins: meant for binding regulators.

<u>Transcription</u> produces molecules (RNA or, through RNA, proteins) that bind to regulatory region of other genes (or that are endproducts). Human (and mammalian) Genome Size 3Gbp (Giga base pairs) 750MB @ 4bp/Byte (CD) Non-repetitive: 1Gbp 250MB In genes: 320Mbp 80MB Coding: 160Mbp 40MB Protein-coding genes: 30,000-40,000 <u>M.Genitalium</u> (smallest true organism) 580,073bp 145KB (eBook) <u>E.Coli</u> (bacteria): 4Mbp 1MB (floppy)

<u>Yeast</u> (eukarya): 12Mbp 3MB (MP3 song) <u>Wheat</u> 17Gbp 4.25GB (DVD)

Gene Composition



Indirect Gene Effects

No combination of standard high-throughput experiments can reconstruct an a-priori known gene/protein network [Wagner].

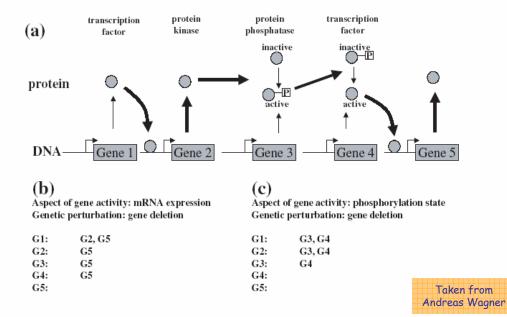
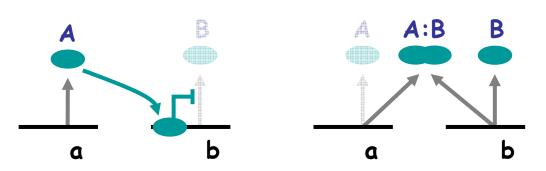
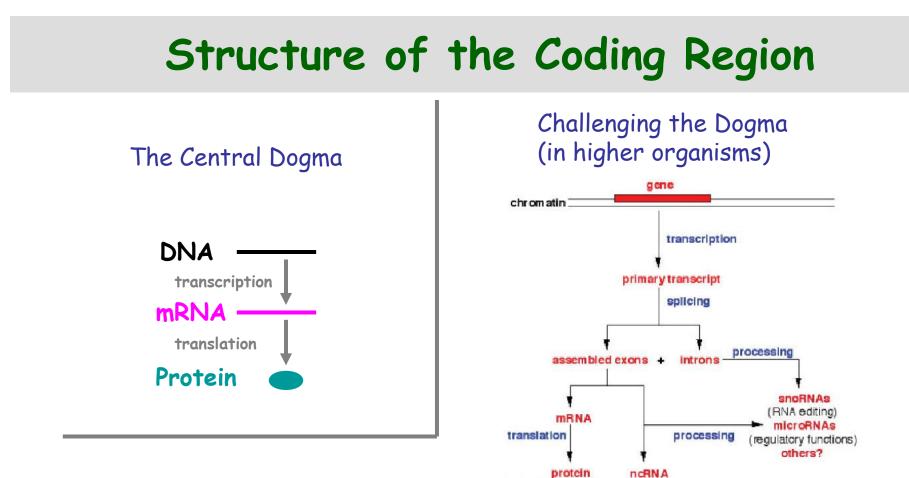


Fig. 1. The importance of specifiying gene activity when reconstructing genetic networks. (a) A hypothetical biochemical pathway involving two transcription factors, a protein kinase, and a protein phosphatase, as well as the genes encoding them. See text for details. (b) Shown is a list of perturbation effects for each of the five genes in (a), when perturbing individual genes by deleting them, and when using mRNA expression level as an indicator of gene activity. The left-most symbol in each line stands for the perturbed gene. To the right of each colon is a list of genes whose activity is affected by the perturbation. (c) Analogous to (b) but for a different notion of gene activity (phosphorylation state).

One of many bistable switches that cannot be described by pure gene regulatory networks [Francois & Hakim].





RNA is not just an intermediary; it can:

- Fold-up like a protein
- Act like an enzyme
- Regulate other transcribed RNA
- Direct protein editing

- ...

97-98% of the transcriptional output of the human genome is non-protein-coding RNA.

(various functions)

(structural, catalytic,

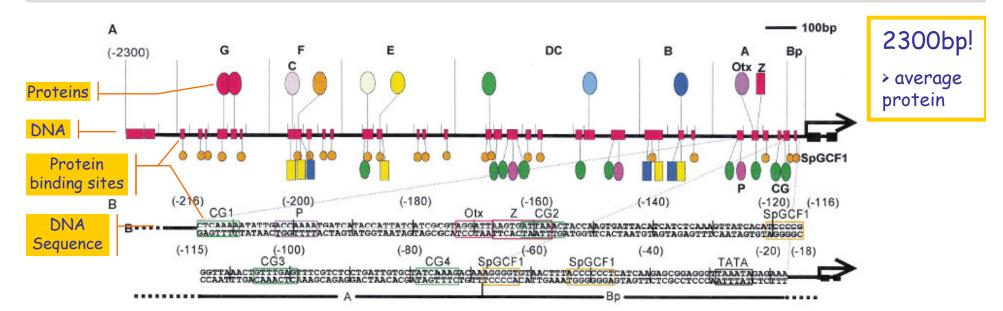
signaling, regulatory)

30-40,000 "protein genes" (1.5% of genome) 60-100,000 "transcription units" (>30% of genome is transcribed)

Taken from

John Mattick

Structure of a Regulatory Region



C Module A functions:

Vegetal plate expression in early development:

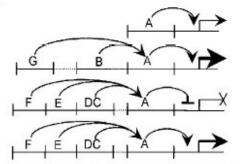
Synergism with modules B and G enhancing endoderm expression in later development:

Repression in ectoderm (modules E and F) and skeletogenic mesenchyme (module DC):



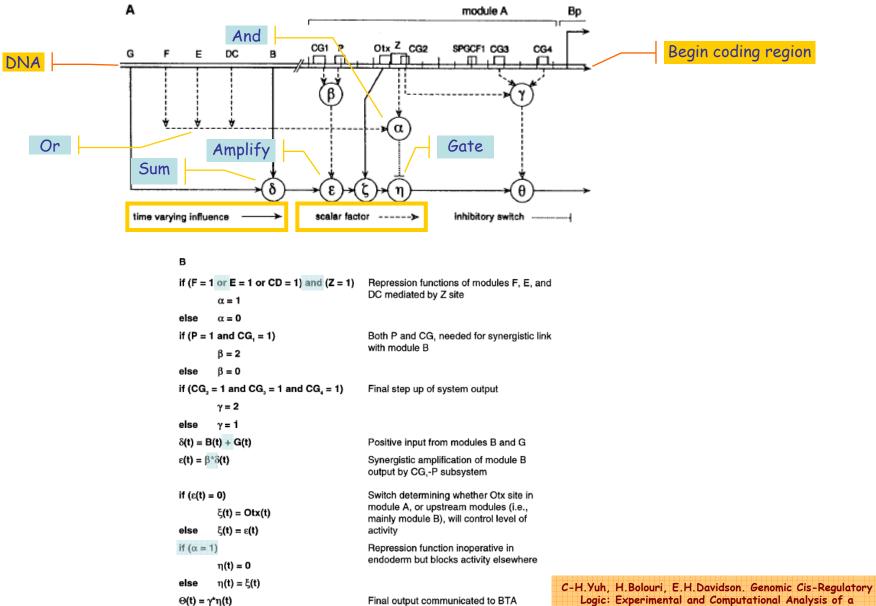
Fig. 1. Endo16 cis-regulatory system and interactive roles of module A. (A) Diversity of protein binding sites and organization into modular subregions [modified from (7)]. Specific DNA binding sites are indicated as red blocks; modular subregions are denoted by letters G to A (Bp, basal promoter). Proteins binding at the target sites considered in this work are indicated: Otx, SpOtx-1 (12); SpGCF1 (14); the proteins CG, Z, and P, which are not yet cloned; and protein C [a CREB family protein (18)] in subregion F. Proteins for which sites occur in multiple regions of the DNA sequence (indicated by the black line) are shown beneath. (B) Sequence of module A and location of protein binding sites. Sites are indicated in the same colors as in (A). A fragment containing CG₃ and CG₄ sites as well as Bp has no endoderm-

specific activity and services other upstream cis-regulatory systems promiscuously; similarly, the *Endo16* cis-regulatory system functions specifically with heterologous promoters substituted for Bp (*5*, *8*, *19*). Boxed sequences indicate conserved core elements of the target sites (*7*, *12*, *14*), not the complete target site sequences. (**C**) Integrative and interactive functions of module A (*5*, *8*). Module A communicates the output of all upstream modules to the basal transcription apparatus. It also initiates endoderm expression, increases the output of modules B and G, and is required for functions of the upstream modules F, E, and DC. These functions are repression of expression in nonendodermal domains and enhancement of expression in response to LiCl.



Taken from Eric H Davidson

Function of a Regulatory Region

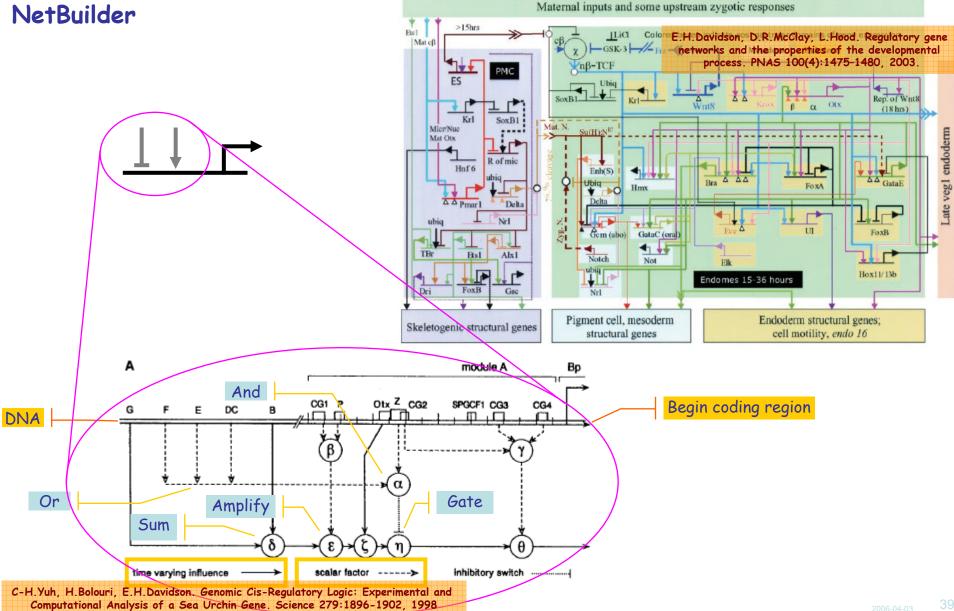


Logic: Experimental and Computational Analysis of a Sea Urchin Gene. Science 279:1896-1902, 1998

Gene Regulatory Networks

http://strc.herts.ac.uk/bio/maria/NetBuilder/

NetBuilder



The Programming Model

- Strange facts about genetic networks:
 - Not an operator algebra. The output of each gate is fixed and pre-determined; it is never a function of the input!
 - Not term-rewriting, nor Petri nets. Inhibition is widespread.
 - Not Communicating Sequential Processes. Feedback is widespread: asynchronous communication needed to avoid immediate self-deadlocks. Even the simplest gates cannot be modeled as a single synchronous automata.
 - Not Message-Passing between genes. Messages themselves have behavior (e.g., they stochastically decay and combine), hence messages are processes as well.
 - Not Data-Flow. Any attempt to use data-flow-style modeling seems doomed because of widespread loops that lead to deadlocks or unbounded queues. Data-flow tokens do not "decay" like proteins.
- How can it possibly work?
 - Stochastic broadcasting. The apparently crude idea of broadcasting a whole bunch of asynchronous decaying messages to activate a future gate, means there are never any "pipeline full" deadlocks, even in presence of abundant feedback loops.
 - Stochastic degradation. Degradation is fundamental for system stability, and at the same time can lead to sudden instability and detection of concentration levels.

Notations for the Gene Machine

- Many of the same techniques as for the Protein Machine apply.
 - Process Calculi, Petri Nets, Term-Rewriting Systems...
- But the "programming model" is different.
 - Asynchronous stochastic control.
 - Biologically poorly understood.
 - Network "motifs" are being analyzed.

- Specific techniques:
 - Hybrid Petri Nets
 - [Matsuno, Doi, Nagasaki, Miyano] Gene Regulation
 - Genomic Object Net www.genomicobject.net
- Gene Regulation Diagrams
- Mixed Gene-Protein Diagrams

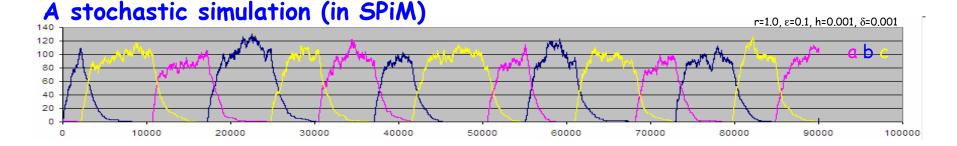
Gene Gates and Circuits

A gene gate
a
neg $neg(a,b) \triangleq$
 $?a_r; \tau_\eta; neg(a,b) +$
 $\tau_{\varepsilon}; (tr(b) | neg(a,b))$
 $tr(p) \triangleq (!p_r; tr(p)) + \tau_{\delta}$ A genetic circuit (engineered in E.Coli) \bigcap_{neg}
b
neg(a,b) |
neg(b,c) |
neg(c,a)

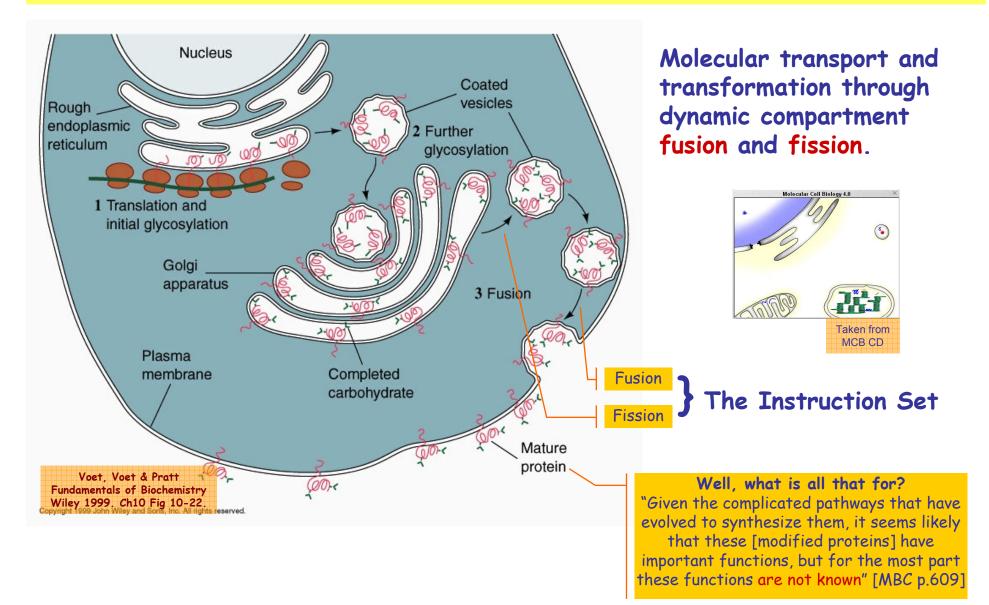
neg

neg

```
The stochastic-\pi program
val dk = 0.001
                 (* Decay rate *)
val inh = 0.001
                 (* Inhibition rate *)
val cst = 0.1
                 (* Constitutive rate *)
let tr(p:chan()) =
   do !p; tr(p) or delay@dk
let neg(a:chan(), b:chan()) =
  do ?a; delay@inh; neg(a,b)
 or delay@cst; (tr(b) | neg(a,b))
(* The circuit *)
val bnd = 1.0
                  (* Protein binding rate *)
new a@bnd:chan() new b@bnd:chan() new c@bnd:chan()
run (neg(c,a) | neg(a,b) | neg(b,c))
```

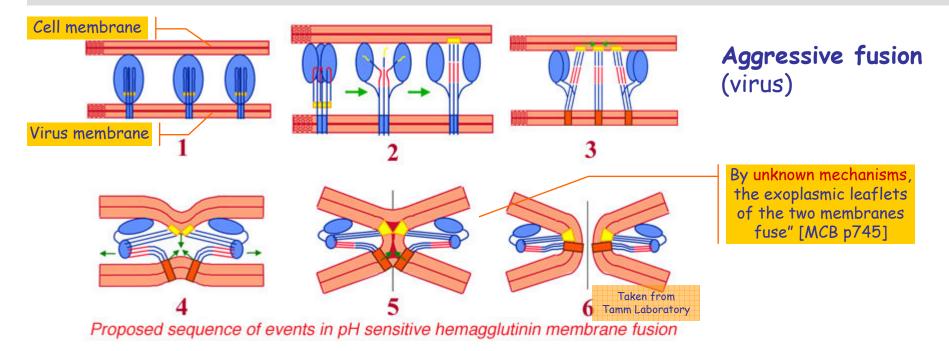


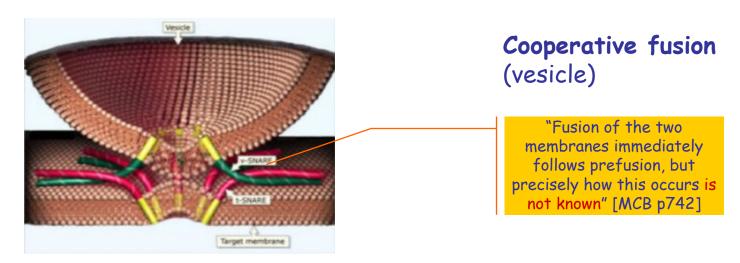
3. The Membrane Machine Very far from the atoms.



Membrane Fusion

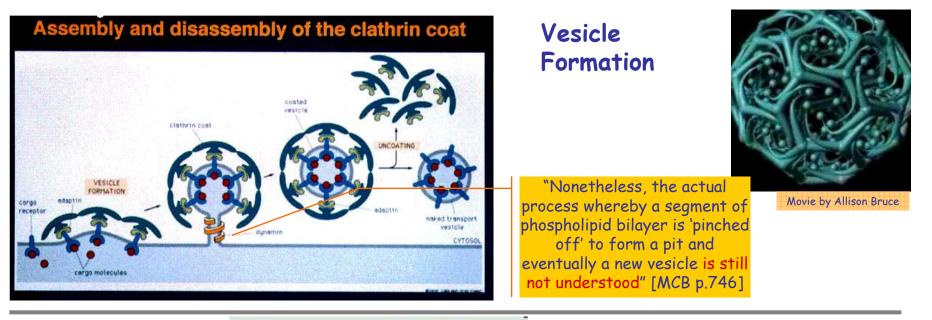
Positive curvature to Negative curvature transition in 3D

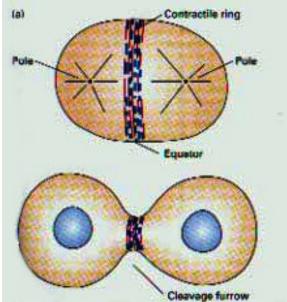




Membrane Fission

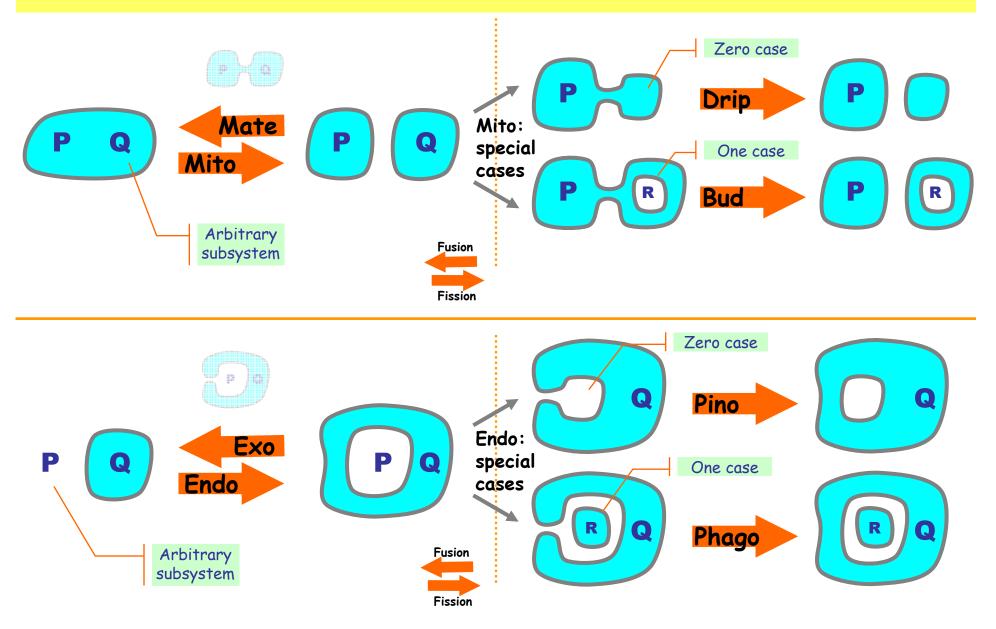
Negative curvature to Positive curvature transition in 3D



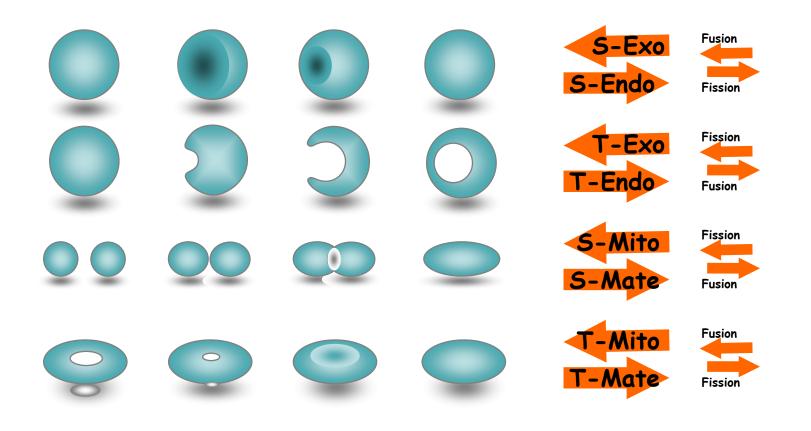


Cytokinesis (Mitosis)

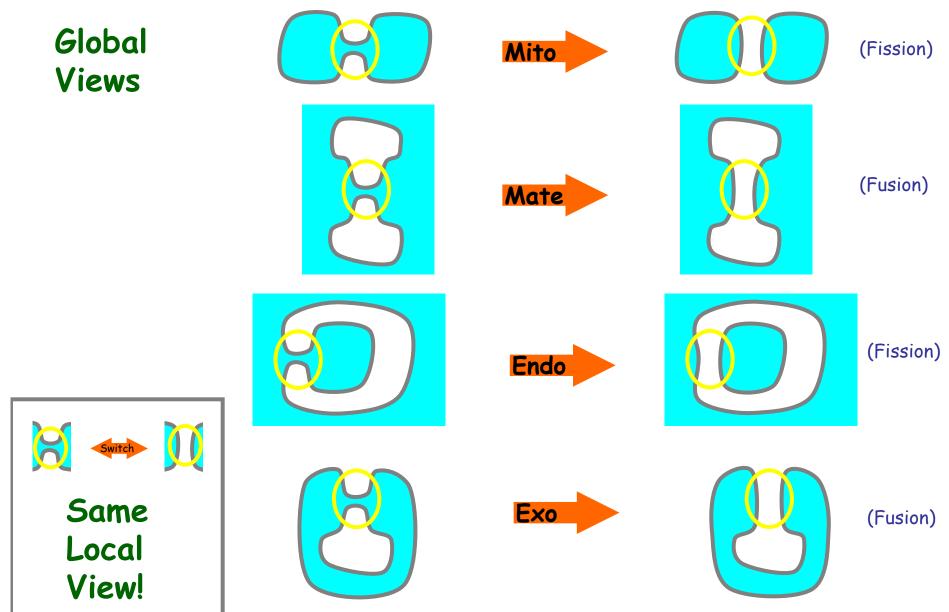
The Membrane Machine "Instruction Set"



... in 3D

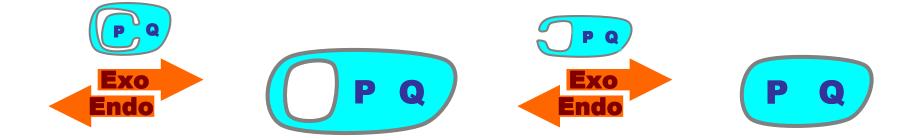


Locally Implementable!



Mito/Mate by 3 Endo/Exo





Notations for the Membrane Machine

- "Snapshot" diagrams
 - In biology literature.
- P-Systems
 - G.Paun uses ideas from the theory of grammars and formal languages to model "Membrane Computing" (book 2002).

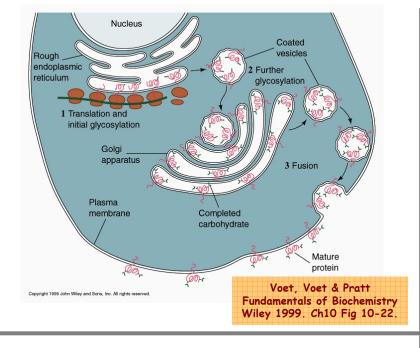
http://psystems.disco.unimib.it/.

• BioAmbients

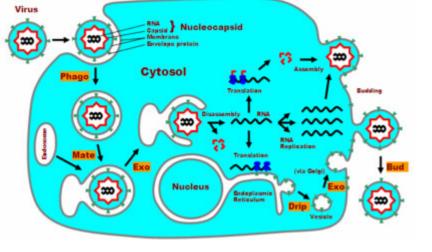
- An extension of BioSPI along Ambient Calculus lines (with more bio-relevant mobility primitives) to model dynamic compartments.
- Brane Calculi
 - Computation on the membrane...

Membrane Algorithms

Protein Production and Secretion

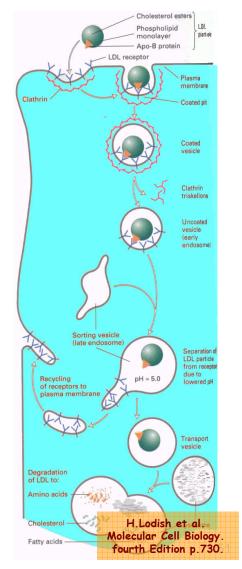


Viral Replication

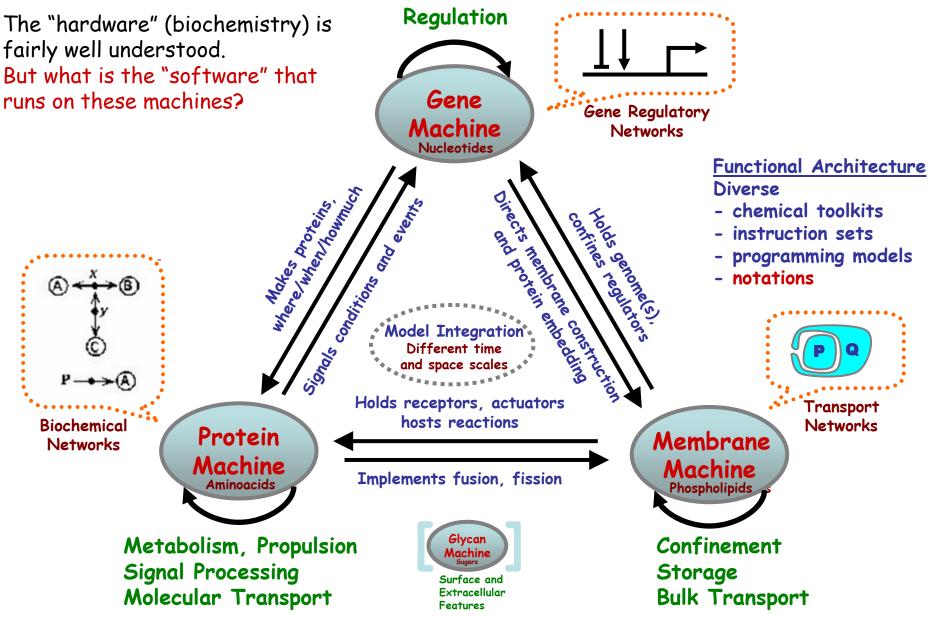


Adapted from: B.Alberts et al. Molecular Biology of the Cell third edition p.279.

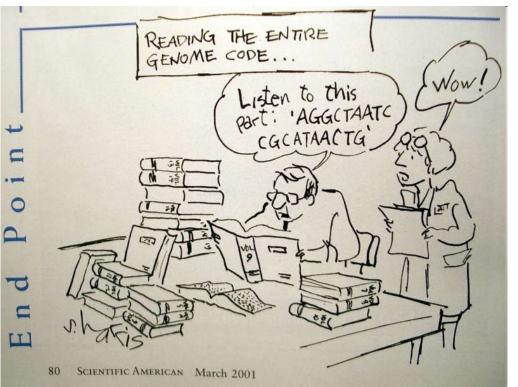




Abstract Machines of Systems Biology



Conclusions



- Q: "The data are accumulating and the computers are humming, what we are lacking are the words, the grammar and the syntax of a new language..." D. Bray (TIBS 22(9):325-326, 1997)
- A: "The most advanced tools for computer process description seem to be also the best tools for the description of biomolecular systems."

E.Shapiro (Lecture Notes)

References

[MCB] Molecular Cell Biology, Freeman. [MBC] Molecular Biology of the Cell, Garland. [Ptashne] A Genetic Switch. [Davidson] Genomic Regulatory Systems.

[Milner] Communicating and Mobile Systems: the Pi-Calculus. [Regev] Computational Systems Biology: A Calculus for Biomolecular Knowledge (Ph.D. Thesis).

Papers

BioAmbients

a stochastic calculus with compartments.

Brane Calculi

process calculi with computation "on" the membranes, not inside them. *Bitonal Systems*

membrane reactions and their connections to "local" patch reactions. *Abstract Machines of Systems Biology*

the abstract machines implemented by biochemical toolkits.

www.luca.demon.co.uk/BioComputing.htm