Types

(Type X) $\dfrac{E', X{<:}A, E'' \vdash \diamond}{E', X{<:}A, E'' \vdash X}$ (Type Top) $\dfrac{E \vdash \diamond}{E \vdash \mathbf{Top}}$

(Type Object) (l_i distinct, $\upsilon_i \in \{^o, ^-, ^+\}$) $\dfrac{E, X{<:}\mathbf{Top} \vdash B_i \quad \forall i \in 1..n}{E \vdash \mathbf{Object}(X)[l_i \upsilon_i : B_i \; ^{i \in 1..n}]}$

(Type Class) (where $A \equiv \mathbf{Object}(X)[l_i \upsilon_i : B_i\{X\} \; ^{i \in 1..n}]$) $\dfrac{E \vdash A}{E \vdash \mathbf{Class}(A)}$

For subtyping, we have:

Subtyping

(Sub Refl) $\dfrac{E \vdash A}{E \vdash A <: A}$ (Sub Trans) $\dfrac{E \vdash A <: B \quad E \vdash B <: C}{E \vdash A <: C}$ (Sub X) $\dfrac{E', X{<:}A, E'' \vdash \diamond}{E', X{<:}A, E'' \vdash X <: A}$ (Sub Top) $\dfrac{E \vdash A}{E \vdash A <: \mathbf{Top}}$

(Sub Object) (where $A \equiv \mathbf{Object}(X)[l_i \upsilon_i : B_i\{X\} \; ^{i \in 1..n+m}]$, $A' \equiv \mathbf{Object}(X')[l_i \upsilon_i' : B_i'\{X'\} \; ^{i \in 1..n}]$)

$$\frac{E \vdash A \quad E \vdash A' \quad E, X{<:}A' \vdash \upsilon_i \, B_i\{X\} <: \upsilon_i' \, B_i'\{\!|A'|\!\} \quad \forall \, i \in 1..n}{E \vdash A <: A'}$$

(Sub Invariant) $\dfrac{E \vdash B}{E \vdash {}^o B <: {}^o B}$ (Sub Covariant) $\dfrac{E \vdash B <: B' \quad \upsilon \in \{^o, ^+\}}{E \vdash \upsilon \, B <: {}^+ B'}$ (Sub Contravariant) $\dfrac{E \vdash B' <: B \quad \upsilon \in \{^o, ^-\}}{E \vdash \upsilon \, B <: {}^- B'}$

The most interesting rule is (Sub Object), which deals with subtypings between object types with variance annotations. It is more complex than the analogous rules that we have considered so far because of the occurrence of a bound variable X in the construct $\mathbf{Object}(X)[\ldots]$. When testing $A <: A'$, component by component, the occurrences of X' in A' are replaced with A'; for the occurrences of X in A, there is the assumption that $X <: A'$. As in Chapter 8, three rules complement (Sub Object) for proving subtypings with variance annotations. In the example of Section 12.3, $CPoint2 <: Point$ is establishing by checking that ${}^+Int \to X <: {}^+Int \to Point$ under the assumption $X <: Point$.

The remaining rules for subtyping are basic ones (such as that for reflexivity). Note in particular that there is no rule for subtyping class types.

Finally, O–1 has the following rules for terms: