(Val Object), for some type $Obj(X)[l_i\upsilon_i{:}B_i\{X\}\ ^{i\in 1..n}]$ such that $E \vdash Obj(X)[l_i\upsilon_i{:}B_i\{X\}\ ^{i\in 1..n}] <: A$. Therefore $E, x_i{:}Obj(X)[l_i\upsilon_i{:}B_i\{X\}\ ^{i\in 1..n}] \vdash b_i : B_i\{\!|Obj(X)[l_i\upsilon_i{:}B_i\{X\}\ ^{i\in 1..n}]|\!\}$ for $i\in 1..n$.

Since we have $E \vdash Obj(X)[l_i\upsilon_i{:}B_i\{X\}\ ^{i\in 1..n}] <: A$, we must also have $E \vdash Obj(X)[l_i\upsilon_i{:}B_i\{X\}\ ^{i\in 1..n}]$, and therefore $E, X<{:}Top \vdash B_i\{X^+\}$ for $i\in 1..n$ and $X\notin dom(E)$. Since $X\notin dom(E)$, we have $X\notin dom(T)$ by Lemma 17.5-2(1). Hence $\emptyset, X<{:}Top \vdash B_i\{X\}\{\leftarrow T\}$ by Lemmas 17.5-2(3) and 17.5-1(2). Take $A^\dagger \equiv Obj(X)[l_i\upsilon_i{:}B_i\{X\}\ ^{i\in 1..n}]\{\leftarrow T\}$. Therefore, since $\emptyset, X<{:}Top \vdash B_i\{X\}\{\leftarrow T\}$, (Method Type) yields $\vDash A^\dagger\Rightarrow j \in Meth$ for $j\in 1..n$.

Take $\Sigma^\dagger \equiv \Sigma, \iota_j\mapsto(A^\dagger\Rightarrow j)\ ^{j\in 1..n}$; by (Store Type) we have $\Sigma^\dagger \vDash \diamond$, because $\iota_j\notin dom(\sigma)$, and hence $\iota_j\notin dom(\Sigma)$, and because $\vDash A^\dagger\Rightarrow j \in Meth$ for $j\in 1..n$.

(1) Since $\Sigma^\dagger$ is an extension of Σ, we also have $\Sigma^\dagger \vDash S\bullet T : E$ by Lemma 17.5-3, so $\Sigma^\dagger \vDash S\bullet\emptyset : E\{\leftarrow T\}$ by Lemma 17.5-2(2). From $E, x_i{:}Obj(X)[l_i\upsilon_i{:}B_i\{X\}\ ^{i\in 1..n}] \vdash b_i : B_i\{\!|Obj(X)[l_i\upsilon_i{:}B_i\{X\}\ ^{i\in 1..n}]|\!\}$ we have $E\{\leftarrow T\}, x_i{:}A^\dagger \vdash b_i : B_i\{\leftarrow T\}\{\!|A^\dagger|\!\}$ by Lemma 17.5-2(3), that is, $E\{\leftarrow T\}, x_i{:}\Sigma^\dagger{}_1(\iota_i) \vdash b_i : \Sigma^\dagger{}_2(\Sigma^\dagger{}_1(\iota_i),\iota_i)$.

(2) We have that σ is of the form $\varepsilon_k\mapsto\langle\varsigma(x_k)b_k,S_k\rangle\ ^{k\in 1..m}$. Now $\Sigma \vDash \sigma$ must come from the (Store Typing) rule, with $\Sigma \vDash S_k\bullet\emptyset : E_k$ and $E_k, x_k{:}\Sigma_1(\varepsilon_k) \vdash b_k : \Sigma_2(\Sigma_1(\varepsilon_k),\varepsilon_k)$. By Lemma 17.5-3, $\Sigma^\dagger \vDash S_k\bullet\emptyset : E_k$; moreover $E_k, x_k{:}\Sigma^\dagger{}_1(\varepsilon_k) \vdash b_k : \Sigma^\dagger{}_2(\Sigma^\dagger{}_1(\varepsilon_k),\varepsilon_k)$, because $\Sigma^\dagger(\varepsilon_k) = \Sigma(\varepsilon_k)$ for $k\in 1..m$ since $dom(\sigma) = dom(\Sigma) = \{\varepsilon_k\ ^{k\in 1..m}\}$ and $\Sigma^\dagger$ extends Σ.

Take $\sigma^\dagger \equiv (\sigma, \iota_i\mapsto\langle\varsigma(x_i)b_i,S\rangle\ ^{i\in 1..n})$. By (1) and (2), via the (Store Typing) rule, we have $\Sigma^\dagger \vDash \sigma^\dagger$. Since $\Sigma^\dagger \vDash \diamond$ and $\Sigma^\dagger \equiv \Sigma, \iota_j\mapsto(A^\dagger\Rightarrow j)\ ^{j\in 1..n}$, we have $\Sigma^\dagger \vDash [l_i{=}\iota_i\ ^{i\in 1..n}] : A^\dagger$ by the (Result Object) rule.

We conclude that $\Sigma^\dagger \geqslant \Sigma \wedge \Sigma^\dagger \vDash \sigma^\dagger \wedge dom(\sigma^\dagger) = dom(\Sigma^\dagger) \wedge \Sigma^\dagger \vDash [l_i{=}\iota_i\ ^{i\in 1..n}] : A^\dagger \wedge \emptyset \vdash A^\dagger <: A\{\leftarrow T\}$.

Case (Red Select)

$$\frac{\sigma\bullet S \vdash a \rightsquigarrow [l_i{=}\iota_i\ ^{i\in 1..n}]\bullet\sigma' \qquad \sigma'(\iota_j) = \langle\varsigma(x_j)b_j,S'\rangle \qquad x_j\notin dom(S') \qquad j\in 1..n \qquad \sigma'\bullet(S', x_j\mapsto[l_i{=}\iota_i\ ^{i\in 1..n}]) \vdash b_j \rightsquigarrow v\bullet\sigma''}{\sigma\bullet S \vdash a.l_j \rightsquigarrow v\bullet\sigma''}$$

By hypothesis $E \vdash a.l_j : A \wedge \Sigma \vDash \sigma \wedge dom(\sigma) = dom(\Sigma) \wedge \Sigma \vDash S\bullet T : E$.

Since $E \vdash a.l_j : A$, we must have $E \vdash a.l_j : B_j\{\!|C|\!\}$ by (Val Select), with $E \vdash B_j\{\!|C|\!\} <: A$, and $E \vdash a : C$ and $E \vdash C <: D$, where D has the form $Obj(X)[l_j\upsilon_j{:}B_j\{X\}, \ldots]$ and $\upsilon_j\in\{^o,^+\}$.

By induction hypothesis, since $E \vdash a : C \wedge \sigma\bullet S \vdash a \rightsquigarrow [l_i{=}\iota_i\ ^{i\in 1..n}]\bullet\sigma' \wedge \Sigma \vDash \sigma \wedge dom(\sigma) = dom(\Sigma) \wedge \Sigma \vDash S\bullet T : E$, there exist a closed type A' and a store type Σ' such that $\Sigma' \geqslant \Sigma \wedge \Sigma' \vDash \sigma' \wedge dom(\sigma') = dom(\Sigma') \wedge \Sigma' \vDash [l_i{=}\iota_i\ ^{i\in 1..n}] : A' \wedge \emptyset \vdash A' <: C\{\leftarrow T\}$.

Since $\sigma'(\iota_j) = \langle\varsigma(x_j)b_j,S'\rangle$, the judgment $\Sigma' \vDash \sigma'$ must come via (Store Typing) from $\Sigma' \vDash S'\bullet\emptyset : E_j$ and $E_j, x_j{:}\Sigma'_1(\iota_j) \vdash b_j : \Sigma'_2(\Sigma'_1(\iota_j),\iota_j)$ for some E_j. Since $\Sigma' \vDash [l_i{=}\iota_i\ ^{i\in 1..n}] : A'$ must come from (Result Object), we have $A' \equiv Obj(X)[l_i\upsilon_i'{:}\Sigma'_2(X,\iota_i)\ ^{i\in 1..n}] \equiv \Sigma'_1(\iota_j)$. Since $E \vdash C <: D$ and $\Sigma \vDash S\bullet T : E$, we obtain $\emptyset \vdash C\{\leftarrow T\} <: D\{\leftarrow T\}$ by Lemma 17.5-2, and $\emptyset \vdash$