Take $A^\dagger \equiv \forall(X{<:}C)B$ and $\Sigma^\dagger \equiv \Sigma$. We conclude $\Sigma^\dagger \geqslant \Sigma \wedge \Sigma^\dagger \vDash \sigma \wedge dom(\sigma) = dom(\Sigma^\dagger) \wedge \Sigma^\dagger \vDash \langle\lambda()b,S\rangle : A^\dagger \wedge \emptyset \vdash A^\dagger <: A\{\leftarrow T\}$.

Case (Red Appl2)

$$\frac{\sigma{\bullet}S \vdash a \rightsquigarrow \langle\lambda()b,S'\rangle{\bullet}\sigma' \qquad \sigma'{\bullet}S' \vdash b \rightsquigarrow v{\bullet}\sigma''}{\sigma{\bullet}S \vdash a() \rightsquigarrow v{\bullet}\sigma''}$$

By hypothesis, $E \vdash a() : A \wedge \Sigma \vDash \sigma \wedge dom(\sigma) = dom(\Sigma) \wedge \Sigma \vDash S{\bullet}T : E$.

Since $E \vdash a() : A$ we must have $E \vdash a() : B\{\!|C|\!\}$ with $E \vdash B\{\!|C|\!\} <: A$ by (Val Appl2<:) for some types C and $\forall(X{<:}D)B\{X\}$ such that $E \vdash a : \forall(X{<:}D)B\{X\}$ and $E \vdash C <: D$. We obtain $\emptyset \vdash C\{\leftarrow T\} <: D\{\leftarrow T\}$, by Lemmas 17.5-2(3) and 17.5-1(2).

By induction hypothesis, since $E \vdash a : \forall(X{<:}D)B \wedge \sigma{\bullet}S \vdash a \rightsquigarrow \langle\lambda()b,S'\rangle{\bullet}\sigma' \wedge \Sigma \vDash \sigma \wedge dom(\sigma) = dom(\Sigma) \wedge \Sigma \vDash S{\bullet}T : E$, there exist a closed type A' and a store type Σ' such that $\Sigma' \geqslant \Sigma \wedge \Sigma' \vDash \sigma' \wedge dom(\sigma') = dom(\Sigma') \wedge \Sigma' \vDash \langle\lambda()b,S'\rangle : A' \wedge \emptyset \vdash A' <: (\forall(X{<:}D)B)\{\leftarrow T\}$.

Now $\Sigma' \vDash \langle\lambda()b,S'\rangle : A'$ must come from (Result Fun2<:), with $A' \equiv \forall(X{<:}D')B'$, $\Sigma' \vDash S'{\bullet}\emptyset : E'$, and $E', X{<:}D' \vdash b : B'$. We have also $\emptyset \vdash D\{\leftarrow T\} <: D'$ and $\emptyset, X{<:}D\{\leftarrow T\} \vdash B' <: B\{\leftarrow T\}$ since $\emptyset \vdash A' <: (\forall(X{<:}D)B)\{\leftarrow T\}$, by Lemma 16.4-5.

Take $T' \equiv \emptyset, X{\mapsto}C\{\leftarrow T\}$. We have $\Sigma' \vDash S'{\bullet}\emptyset : E'$ and $\emptyset \vdash C\{\leftarrow T\} <: D'$ by transitivity. Therefore, $\Sigma' \vDash S'{\bullet}T' : (E', X{<:}D')$ by (Stack X Typing).

By induction hypothesis, since $E', X{<:}D' \vdash b : B' \wedge \sigma'{\bullet}S' \vdash b \rightsquigarrow v{\bullet}\sigma'' \wedge \Sigma' \vDash \sigma' \wedge dom(\sigma') = dom(\Sigma') \wedge \Sigma' \vDash S'{\bullet}T' : (E', X{<:}D')$, there exist a closed type $A^\dagger$ and a store type $\Sigma^\dagger$ such that $\Sigma^\dagger \geqslant \Sigma' \wedge \Sigma^\dagger \vDash \sigma'' \wedge dom(\sigma'') = dom(\Sigma^\dagger) \wedge \Sigma^\dagger \vDash v : A^\dagger \wedge \emptyset \vdash A^\dagger <: B'\{\leftarrow T'\}$.

We have $(B'\{X\})\{\leftarrow T'\} \equiv B'\{\!|C\{\leftarrow T\}|\!\}$ by definition of T', hence $\emptyset \vdash A^\dagger <: B'\{\!|C\{\leftarrow T\}|\!\}$. Since $\emptyset \vdash C\{\leftarrow T\} <: D\{\leftarrow T\}$ and $\emptyset, X{<:}D\{\leftarrow T\} \vdash B'\{X\} <: B\{X\}\{\leftarrow T\}$, we have $\emptyset \vdash B'\{\!|C\{\leftarrow T\}|\!\} <: B\{\!|C\{\leftarrow T\}|\!\}\{\leftarrow T\}$ by Lemma 17.5-1. Since $E \vdash B\{\!|C|\!\} <: A$, we have $\emptyset \vdash (B\{\!|C|\!\})\{\leftarrow T\} <: A\{\leftarrow T\}$ by Lemmas 17.5-2(3) and 17.5-1(2). Finally $\emptyset \vdash A^\dagger <: A\{\leftarrow T\}$ by transitivity.

We conclude $\Sigma^\dagger \geqslant \Sigma \wedge \Sigma^\dagger \vDash \sigma'' \wedge dom(\sigma'') = dom(\Sigma^\dagger) \wedge \Sigma^\dagger \vDash v : A^\dagger \wedge \emptyset \vdash A^\dagger <: A\{\leftarrow T\}$.

□

We have also an analogue to Corollary 11.4-4:

Corollary 17.5-5

If $\emptyset \vdash a : A$ and $\emptyset{\bullet}\emptyset \vdash a \rightsquigarrow v{\bullet}\sigma$, then there exist a type $A^\dagger$ and a store type $\Sigma^\dagger$ such that $\Sigma^\dagger \vDash \sigma$ and $\Sigma^\dagger \vDash v : A^\dagger$, with $\emptyset \vdash A^\dagger <: A$.

□

This corollary implies, for example, that if $\emptyset \vdash a : A$ and $\emptyset{\bullet}\emptyset \vdash a \rightsquigarrow v{\bullet}\sigma$ where A is an object type, then v is an object result of the form $[l_i{=}\varsigma_i{}^{i\in 1..n}]$ and never a type abstraction result of the form $\langle\lambda()b,S\rangle$.